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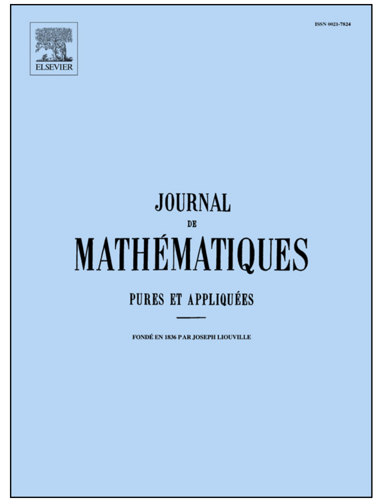

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Convergence order of upwind type schemes for transport equations with discontinuous coefficients

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Abstract

An analysis of the error of the upwind scheme for transport equations with discontinuous coefficients is provided. We consider here a velocity field that is bounded and one-sided Lipschitz continuous. In this framework, solutions are defined in the sense of measures along the lines of Poupaud and Rascle's work. We study the convergence order of the upwind scheme for any Wasserstein distance and we prove that this order is 1/2. We also establish the optimality of this rate of convergence. In the appendix, we show that this result also applies to other diffusive first order finite volume schemes and to a forward semi-Lagrangian scheme.

Résumé

Cet article propose une analyse de l'erreur du schéma décentré *amont* pour des équations de transport à coefficients discontinus. Nous considérons un champ de vitesse borné et lipschitzien à droite. Dans ce contexte, les solutions sont définies au sens des mesures, à la manière de Poupaud et Rascle. Nous étudions l'ordre de convergence du schéma en distance de Wasserstein et démontrons qu'il vaut 1/2. Nous montrons que cet ordre de convergence est optimal. Dans l'annexe, nous montrons que ce résultat s'étend à d'autres schémas de volumes finis, diffusifs et d'ordre 1, et à un schéma de type semi-lagrangien en avant.

Keywords: upwind finite volume scheme, forward semi-Lagrangian scheme, convergence order, conservative transport equation, continuity equation, measure-valued solution.
2010 MSC: 35D30, 35L65, 65M12, 65M15.

1. Introduction

This paper is devoted to the numerical analysis of an upwind scheme for the linear transport equation in conservative form (continuity equation) with discontinuous coefficients. In space dimension d, this equation reads

$$\partial_t \rho + \operatorname{div}\left(a\rho\right) = 0, \qquad t > 0, \quad x \in \mathbb{R}^d, \tag{1.1}$$

and is complemented with the initial condition $\rho(0,\cdot) = \rho^{ini}$.

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