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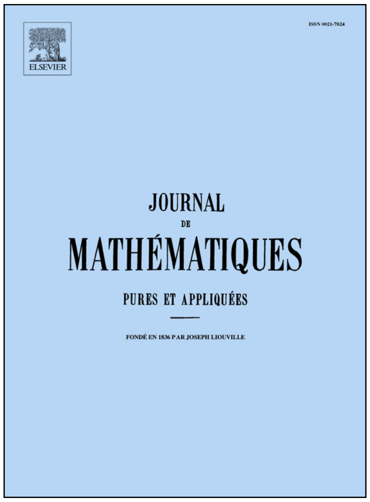

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Super-linear spreading in local and non-local cane toads equations

Emeric Bouin [*] Christopher Henderson [†] Lenya Ryzhik [‡]

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Abstract

In this paper, we show super-linear propagation in a nonlocal reaction-diffusion-mutation equation modeling the invasion of cane toads in Australia that has attracted attention recently from the mathematical point of view. The population of toads is structured by a phenotypical trait that governs the spatial diffusion. In this paper, we are concerned with the case when the diffusivity can take unbounded values, and we prove that the population spreads as $t^{3/2}$. We also get the sharp rate of spreading in a related local model.

1 Introduction

The invasion of cane toads in Australia has interesting features different from the standard spreading observed in most other species. The experimental data [39, 42] show that the invasion speed has steadily increased during the eighty years since the toads were introduced in Australia. In addition, the younger individuals at the edge of the invasion front have a significantly different morphology compared to other populations – their legs tend to be on average much longer than away from the front. This is just one example of a non-uniform space-trait distribution – see, for instance, a study on the expansion of bush crickets in Britain [43]. Several works have addressed the front invasions in ecology, where the trait is related to the dispersal ability [3, 16]. It has been observed that selection of more mobile individuals can occur, even if they have no advantage in their reproductive rate, due to the spatial sorting [1, 33, 39, 40].

In this paper, we focus on the super-linear in time propagation in a model of the cane toads invasion proposed in [5], based on the classical Fisher-KPP equation [22, 34]. The population density is structured by a spatial variable, $x \in \mathbb{R}$, and a motility variable $\theta \in \Theta \overset{\text{def}}{=} [\underline{\theta}, \infty)$, with a fixed $\underline{\theta} > 0$. This population undergoes diffusion in the trait variable θ, with a constant diffusion coefficient $\alpha > 0$, representing mutation, and in the spatial variable, with the diffusion coefficient θ, representing the effect of the trait on the spreading rates of the species. Thus, neglecting the competition and reproduction, the population model for the population density $u(t, x, \theta)$ would be

$$u_t = \theta u_{xx} + \alpha u_{\theta\theta}. \tag{1.1}$$

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[*] CEREMADE - Université Paris-Dauphine, UMR CNRS 7534, Place du Maréchal de Lattre de Tassigny, 75775 Paris Cedex 16, France. E-mail: `bouin@ceremade.dauphine.fr`

[†] Ecole Normale Supérieure de Lyon, UMR CNRS 5669 'UMPA', 46 allée d'Italie, F-69364 Lyon cedex 07, France. E-mail: `christopher.henderson@ens-lyon.fr`

[‡] Department of Mathematics, Stanford University, Stanford, CA 94305, E-mail: `ryzhik@math.stanford.edu`