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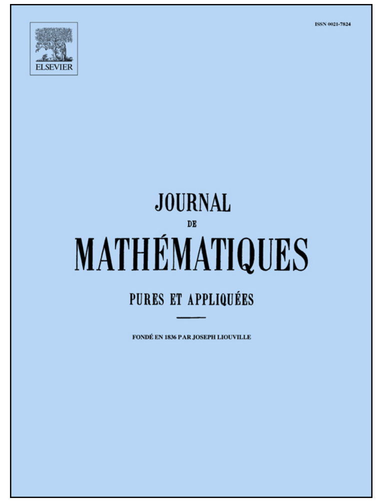

Toric chordality

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TORIC CHORDALITY

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ABSTRACT. We study the geometric change of Chow cohomology classes in projective toric varieties under the Weil-McMullen dual of the intersection product with a Lefschetz element. Based on this, we introduce toric chordality, a generalization of graph chordality to higher skeleta of simplicial complexes with a coordinatization over characteristic 0, leading us to a far-reaching generalization of Kalai's work on applications of rigidity of frameworks to polytope theory. In contrast to "homological" chordality, the notion that is usually studied as a higher-dimensional analogue of graph chordality, we will show that toric chordality has several advantageous properties and applications.

- Most strikingly, we will see that toric chordality allows us to introduce a higher version of Dirac's propagation principle.
- Aside from the propagation theorem, we also study the interplay with the geometric properties of the simplicial chain complex of the underlying simplicial complex, culminating in a quantified version of the Stanley–Murai–Nevo generalized lower bound theorem.
- Finally, we apply our technique to give a simple proof of the generalized lower bound theorem in polytope theory and
- prove the balanced generalized lower bound conjecture of Klee and Novik.

INTRODUCTION

A notion at the very core of graph theory, chordality is a statement about the geometry and complexity of cycles in a graph, stating in essence that a cycle is decomposable in the most economic way imaginable. The relation of graph chordality to commutative algebra in particular has motivated many to attempt a generalization of graph chordality to higher dimensions, often using a homological or combinatorial perspective (we surveyed and summarized this perspective in [ANS16b]). However, homological notions, as we shall argue here, are not adequate to capture graph chordality in simplicial complexes, especially not in relation to commutative algebra, and

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