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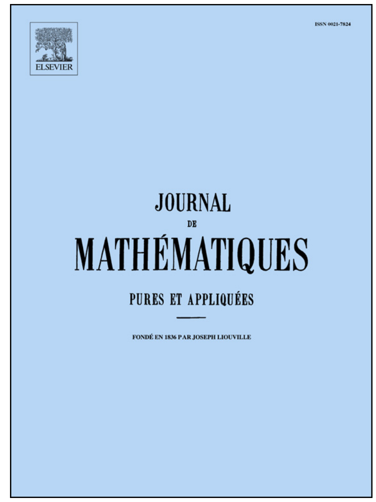

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Parabolic models for chemotaxis on weighted networks

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Abstract

In this article we consider the Keller-Segel model for chemotaxis on networks, both in the doubly parabolic case and in the parabolic-elliptic one. Introducing appropriate transition conditions at vertices, we prove the existence of a time global and spatially continuous solution for each of the two systems. The main tool we use in the proof of the existence result are optimal decay estimates for the fundamental solution of the heat equation on a weighted network.

Keywords: Chemotaxis, network, transmission conditions, heat kernel
2000 MSC: 92C17, 92C42, 35R02, 35Q92, 35A01

1. Introduction

We consider the classical Keller-Segel system for chemotaxis

$$\begin{aligned} u_t &= \Delta u - \nabla \cdot (u\,\nabla c) \\ \varepsilon c_t &= \Delta c + u - \alpha\, c \end{aligned} \tag{1.1}$$

on a finite weighted network Γ, where ε, $\alpha \geq 0$.

System (1.1) has been introduced in the early seventies in [12, 13] in order to model the aggregation phenomenon undergone by the slime mold *Dictyostelium discoideum*. In this biological context, u represents the cell concentration of the organism and satisfies the continuity equation in (1.1), while c is the chemo-attractant concentration and solves the diffusion equation in (1.1). In the Euclidean case, i.e. when (1.1) is considered on a domain of $\mathbb{R}^d$, there is a vast literature on system (1.1). Depending on the space dimension d, $\varepsilon > 0$ (double parabolic case) or $\varepsilon = 0$ (parabolic-elliptic case) and the initial data u^0, c^0, different phenomena can occur: global existence, finite or infinite time blow-up, peaks formation, threshold phenomena, etc. We refer to [6, 10, 22] and the references therein for more details on that problem.

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