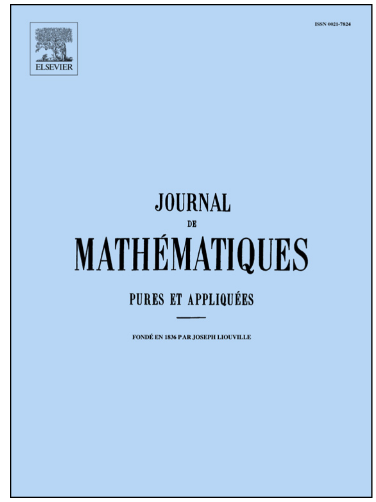

On the moving plane method for singular solutions to semilinear elliptic equations

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ON THE MOVING PLANE METHOD FOR SINGULAR SOLUTIONS TO SEMILINEAR ELLIPTIC EQUATIONS

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Abstract. *We consider positive weak solutions to* $-\Delta u = f(x,u)$ *in* $\Omega \setminus \Gamma$ *with* $u = 0$ *on* $\partial\Omega$. *We prove symmetry and monotonicity properties of the solutions in symmetric convex domains via the moving plane method, under suitable assumptions on* f *and on the singular set* Γ. *With similar arguments we also consider the case when the domain is the whole space and the nonlinearity has at most critical growth.*

Résumé. *Nous considérons les solutions faibles et positives de* $-\Delta u = f(x,u)$ *dans* $\Omega \setminus \Gamma$ *avec* $u = 0$ *sur* $\partial\Omega$. *A l'aide de la méthode des hyperplans mobiles, et sous des hypothèses appropriées sur* f *et sur l'ensemble singulier* Γ, *nous démontrons des propriétés de symétrie et de monotonie pour les solutions définies sur des domaines convexes et symétriques. Avec des arguments similaires, nous considérons également le cas ou le domaine est l'espace tout entier et la fonction non linéaire* f *est à croissance au plus critique.*

1. INTRODUCTION

We consider the problem

$$
\begin{cases} -\Delta u = f(x,u) & \text{in} \quad \Omega \setminus \Gamma \\ u > 0 & \text{in} \quad \Omega \setminus \Gamma \\ u = 0 & \text{on} \quad \partial\Omega \end{cases} \tag{1.1}
$$

where Ω is a bounded smooth domain of $\mathbb{R}^n$ with $n \geq 2$ which is convex in the x_1-direction and symmetric with respect to the hyperplane $\{x_1 = 0\}$. The solution has a possible singularity on the critical set $\Gamma \subset \Omega$ and thus is understood in the following meaning: $u \in H^1_{loc}(\overline{\Omega} \setminus \Gamma) \cap C(\overline{\Omega} \setminus \Gamma)$ and

$$
\int_\Omega \nabla u \nabla \varphi \, dx = \int_\Omega f(x,u)\varphi \, dx \qquad \forall \varphi \in C^1_c(\Omega \setminus \Gamma)\,. \tag{1.2}
$$

The literature regarding singular solutions is really wide and we apologize to the reader that will not find appropriate references here. We suggest the reading of the book of Véron [13] and the references therein.

Here we exploit the moving plane technique (see [1, 7, 11]) in order to obtain symmetry and monotonicity properties of the solutions to (1.1). The technique that we use is close to the one developed in [3] and we also borrow some ideas from [12]. The main feature is the use of the moving plane procedure basing only on the weak maximum principle in small domains. This is not straightforward in our case since solutions are singular on Γ and in particular are not in $H^1_0(\Omega)$. Our main result is the following:

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