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I. Foliation of the physical cylinder

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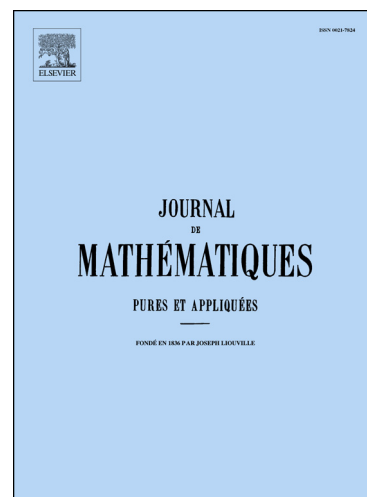
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Lee-Yang zeros for the DHL and 2D rational dynamics, I. Foliation of the physical cylinder.

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Abstract

In a classical work of the 1950's, Lee and Yang proved that the zeros of the partition functions of a ferromagnetic Ising model always lie on the unit circle. Distribution of these zeros is physically important as it controls phase transitions in the model. We study this distribution for the Migdal-Kadanoff Diamond Hierarchical Lattice (DHL). In this case, it can be described in terms of the dynamics of an explicit rational function $\mathcal{R}$ in two variables (the renormalization transformation). We prove that $\mathcal{R}$ is partially hyperbolic on an invariant cylinder C . The Lee-Yang zeros are organized in a transverse measure for the central-stable foliation of $\mathcal{R}|_C$. Their distribution is absolutely continuous. Its density is C^∞ (and non-vanishing) below the critical temperature. Above the critical temperature, it is C^∞ on a open dense subset, but it vanishes on the complementary set of positive measure.

Résumé

Dans un papier célèbre des années 1950, Lee et Yang ont montré que les zéros de la fonction de partition du modèle d'Ising ferromagnétique sont situés sur le cercle unité. La répartition de ces zéros est physiquement intéressante car elle permet de prévoir les transitions de phases de ce modèle. Nous nous proposons d'étudier cette répartition dans le cas particulier du réseau auto-similaire en losange de Migdal-Kadanoff (DHL). Pour ce modèle, la répartition peut être obtenue à travers l'étude des propriétés dynamiques d'une fonction rationnelle explicite de deux variables, $\mathcal{R}$ (la fonction de renormalisation). Nous démontrons que $\mathcal{R}$ est partiellement hyperbolique sur un cylindre invariant C . Nous montrons ensuite que les zéros de Lee et Yang sont répartis selon une mesure transverse pour le feuilletage central et stable de $\mathcal{R}|_C$, et que cette répartition est absolument continue. La densité obtenue est C^∞ (et non-nulle) en dessous de la température critique. Au-dessus de la température critique, elle est C^∞ sur un ouvert dense, et s'annule sur le complémentaire (de mesure positive) de cet ensemble.

Keywords: Ising model, Lee-Yang Zeros, Hierarchical Lattice, Partially Hyperbolic Dynamics.

2010 MSC: 82B20, 82B28, 37D30, 37F10.

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