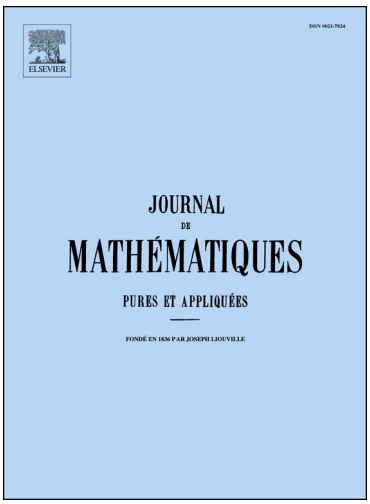

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PII: S0021-7824(16)30084-8
DOI: http://dx.doi.org/10.1016/j.matpur.2016.07.010
Reference: MATPUR 2860

To appear in: *Journal de Mathématiques Pures et Appliquées*

Received date: 27 February 2016

Please cite this article in press as: O. Rossi et al., Shape maps for second order partial differential equations, *J. Math. Pures Appl.* (2016), http://dx.doi.org/10.1016/j.matpur.2016.07.010

Shape maps for second order partial differential equations

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Abstract

We analyse the singularity formation of congruences of solutions of systems of second order PDEs via the construction of *shape maps*. The trace of such maps represents a congruence volume whose collapse we study through an appropriate evolution equation, akin to Raychaudhuri's equation. We develop the necessary geometric framework on a suitable jet space in which the shape maps appear naturally associated with certain linear connections. Explicit computations are given, along with a nontrivial example.

Resumé

Nous analysons la formation de la singularité des congruences de solutions des systèmes des équations aux dérivées partielles du second ordre via la construction de *shape fonction*. La trace de ces fonctions représente un volume de congruence dont l'effondrement nous étudions par une équation d'évolution appropriée, semblable à l'équation de Raychaudhuri. Nous développons le cadre géométrique nécessaire sur un espace de jet approprié dans lequel les shape fonctions apparaissent naturellement comme des objets associes à certaines connexions linéaires. Les calculs explicites sont donnés, avec un exemple non trivial.

Keywords: singularity formation, second order PDEs, connection, curvature, Raychaudhuri's equation
2010 MSC: 58J60, 58A20, 35G50, 35N99

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