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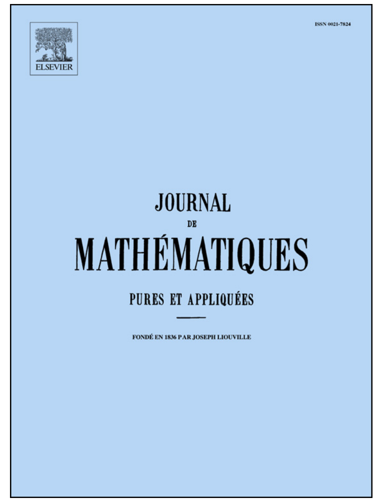

Symmetry breaking for a problem in optimal insulation

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Symmetry breaking for a problem in optimal insulation

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Abstract

We consider the problem of optimally insulating a given domain Ω of $\mathbb{R}^d$; this amounts to solve a nonlinear variational problem, where the optimal thickness of the insulator is obtained as the boundary trace of the solution. We deal with two different criteria of optimization: the first one consists in the minimization of the total energy of the system, while the second one involves the first eigenvalue of the related differential operator. Surprisingly, the second optimization problem presents a symmetry breaking in the sense that for a ball the optimal thickness is nonsymmetric when the total amount of insulator is small enough. In the last section we discuss the shape optimization problem in which Ω is allowed to vary too.

RÉSUMÉ : Nous considérons le problème d'isolation optimale d'un ensemble Ω de $\mathbb{R}^d$; ceci revient à résoudre un problème variationnel non linéaire, où l'épaisseur optimale du matériau isolant est obtenue comme trace au bord de la solution. Nous étudions deux critères différents d'optimalité: le premier consiste dans la minimisation de l'énergie totale du système et le deuxième implique la première valeur propre de l'opérateur associé. De façons surprenante, le deuxième problème présente une brisure de symétrie, dans le sens que pour une boule, l'épaisseur optimale n'est pas symétrique quand la quantité totale du matériau isolant est petite. Dans la dernière section, nous discutons un problème d'optimisation de forme associé, dans lequel l'ensemble Ω est aussi supposé variable.

Keywords: optimal insulation, symmetry breaking, Robin boundary conditions
2010 Mathematics Subject Classification: 49J45, 35J25, 35B06, 49R05

1 Introduction

In the present paper we deal with the problem of determining the best distribution of a given amount of insulating material around a fixed domain Ω of $\mathbb{R}^d$ which represents a thermally conducting body; the thickness of the insulating material is assumed very small with respect to the size of Ω, so the material density is assumed to be a nonnegative function defined on the boundary $\partial\Omega$. A rigorous approach is to consider a limit problem when the thickness of the insulating layer goes to zero and simultaneously the conductivity in the layer goes to zero; this has been studied in [2], [1] (see also [7], [10], [11]), where the family of functionals

$$F_\varepsilon(u) = \frac{1}{2}\int_\Omega |\nabla u|^2\,dx + \frac{\varepsilon}{2}\int_{\Sigma_\varepsilon} |\nabla u|^2\,dx - \int_\Omega fu\,dx$$

is considered on the Sobolev space $H^1_0(\Omega \cup \Sigma_\varepsilon)$, and where Σ_ε is a thin layer of variable thickness $\varepsilon h(\sigma)$ around the boundary $\partial\Omega$

$$\Sigma_\varepsilon = \big\{\sigma + t\nu(\sigma)\ :\ \sigma \in \partial\Omega,\ 0 \le t < \varepsilon h(\sigma)\big\}.$$