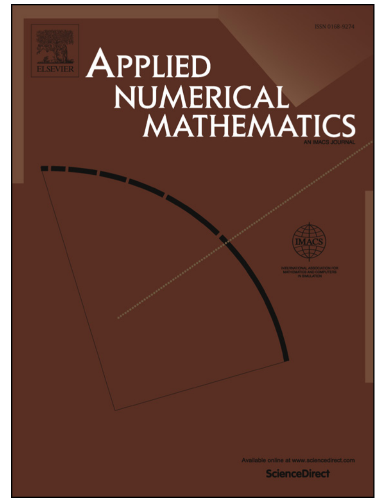

A spectral framework for fractional variational problems based on fractional Jacobi functions

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A spectral framework for fractional variational problems based on fractional Jacobi functions[☆]

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Abstract

A family of orthogonal systems of fractional functions is developed. The orthogonal systems are based on Jacobi polynomials through a fractional coordinate transform. This family of orthogonal systems offers great flexibility to match a wide range of fractional differential models. Approximation errors by the basic orthogonal projection are established. New three kinds of fractional Jacobi-Gauss-type interpolation are introduced. As an example of applications, an efficient approximation based on the proposed fractional functions to a fractional variational problem is presented and implemented. This approximation takes into account the potential irregularity of the solution, and so we are able to obtain a result on optimal order of convergence without the need to impose inconvenient smoothness conditions on the solution. Implementation details are provided for the scheme, together with a series of numerical examples to show the efficiency of the proposed method.

Keywords: Fractional Jacobi polynomials; Fractional variational problems; Müntz-Legendre-Gauss-type quadrature; Fractional optimal control.
MSC: 65N35; 65D32; 65K10; 49M05

1. Introduction

The basic idea behind fractional calculus has a history aligned with that of more classical calculus and the topic has attracted the interest of mathematicians who contributed fundamentally to the development of classical calculus, including L'Hospital, Leibniz, Riemann, Liouville, Grünward, and Letnikov. Progress in the last two decades has demonstrated that many systems in science and engineering can be modeled more accurately by employing fractional-order rather than integer-order derivatives [1–3]. It is important to point out that, unlike the integer derivatives which are locally defined on the epsilon neighborhood of a chosen point, the fractional derivatives are globally defined by a definite integral over the whole domain.

Numerical computation of the fractional integrals and derivatives is the key to understand fractional calculus and solve fractional differential equations of increasing interest in many fields of science and engineering, see [4, 5]. However, the development of numerical schemes in this area does not have a long history and has recently undergone a fast evolution. The majority of numerical methods for fractional differential equations impose some unreasonable restrictions on the solutions such that high accuracy can be achieved. For example, the Grünwald-Letnikov formula [6, 7], the weighted shifted Grünwald-Letnikov formulas [8], the L1 method and its modification [9, 10], the fractional linear multi-step methods [11], the fractional central difference methods [12], and the spectral approximations [13, 14] require that the solution of the considered fractional differential equation is sufficiently smooth such that the expected accuracy can be realized. In particular, most of works related to the latter approach are polynomial-based. It is known

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