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Modulus-based matrix splitting algorithms for the quasi-complementarity problems ☆

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In this paper, a class of modulus-based matrix splitting iteration methods for the quasi-complementarity problems is presented. The convergence analysis of the proposed methods is discussed. Numerical experiments show that the proposed methods are efficient.

1. Introduction

As is known, it is necessary to compute solutions of complementarity problem in engineering and economic applications, such as the linear and quadratic programming, the economies with institutional restrictions upon prices, the optimal stopping in Markov chain and the free boundary problems; see [12,13,17,32,37].

For the linear complementarity problems (LCP), it is to find $z \in \mathbb{R}^n$ such that

$$Az + q \geq 0,\ z \geq 0,\ z^T(Az + q) = 0,\ \text{with } A \in \mathbb{R}^{n\times n} \text{ and } q \in \mathbb{R}^n, \tag{1.1}$$

there are many efficient algorithms to obtain its numerical solution, such at the pivot algorithms [24], the projected successive overrelaxation (SOR) iteration methods [14], the general fixed-point iteration methods [2,31,36,39], the multisplitting methods [5,7,8]. Recently, a class of modulus iteration methods for solving the LCP is very popular, see in [32,40] for its original version, and see [6,9,10,15,18,19,26,27,41,42,46–49] for its other versions. The modulus-method has been applied in many fields, including the solution of least-square problems [11,50].

The LCP becomes the nonlinear complementarity problems (NCP) when using $F(z)$ instead of $Az + q$ in (1.1), where $F(z)$: $\mathbb{R}^n \to \mathbb{R}^n$ is a given function. To obtain the numerical solution of the NCP, some numerical methods have also been developed. These include the asynchronous parallel nonlinear accelerated overrelaxation methods [4], the parallel nonlinear multisplitting relaxation methods [3], the penalty method [22], the smoothing quasi-Newton method [29], two-level additive Schwarz algorithms [45]. Using $F(z)$ instead of q in (1.1), the LCP becomes a class of weakly nonlinear complementarity problems (WNCP), see [28]. For the numerical solution to the WNCP, a variety of iteration schemes have been developed,

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such as the multisplitting iteration methods in [28], the modified semismooth Newton method [38], the modulus-based matrix splitting methods [21,30,43,44], and so on.

The LCP becomes a class of implicit complementarity problems (ICP) when using $z - m(z) \geq 0$ instead of $z \geq 0$ in (1.1), where $m(z)$ is an invertible mapping from $\mathbb{R}^n$ into itself. For finding solutions of the ICP, there exist some effective methods, such as isotonicity of the metric projection [1], the inexact newton method [23]. Recently, Hong and Li in [20] has extended the modulus-based matrix splitting methods to solve a class of implicit complementarity problems (ICP).

As a general case, the quasi-complementarity problems (QCP) in [34] not only includes the linear complementarity problems, but also includes the nonlinear complementarity problems and the implicit complementarity problems. The QCP is of the form: for a given matrix $A \in \mathbb{R}^{n\times n}$ and a vector $q \in \mathbb{R}^n$, a point-to-point mapping Φ and a nonlinear transformation Ψ from $\mathbb{R}^n$ into itself, the QCP(q, A) is to find $z \in \mathbb{R}^n$ satisfy

$$w = Az + q + \Psi(z) \geq 0,\ z - \Phi(z) \geq 0 \text{ and } (z - \Phi(z))^T w = 0. \tag{1.2}$$

Obviously, the QCP (1.2) is the most general and unifying one, which is the main motivation of this paper. Recently, other forms of the QCP have been considered, such as the generalized quasi-complementarity problems [33], the vector implicit quasi-complementarity problems [25], the mixed quasi-complementarity problems [16], the nonlinear quasi-complementarity problems [27,35], and so on.

In this paper, based on the promising performance and elegant mathematical properties of the modulus methods, we extend the modulus-based matrix splitting iteration methods to solve the QCP (1.2). The main scheme of this paper is to obtain an equivalent fixed point equation of the QCP (1.2) by using the change of variables and then solve this fixed point equation by using modulus-based matrix splitting iteration method.

This paper is organized as follows. Some necessary definitions are reviewed in Section 2. A class of modulus-based matrix splitting iteration methods is established and its convergence conditions are discussed in Section 3. Numerical experiments are reported in Section 4, and finally some concluding remarks are given in Section 5.

2. Preliminaries

Some necessary definitions and notations are reviewed in this section, which are used in the sequel discussions.

Let $A = (a_{ij})$ and $B = (b_{ij})$ be two real $n \times n$ matrices, we write $A \geq B$ $(A > B)$ if $a_{ij} \geq b_{ij}$ $(a_{ij} > b_{ij})$ for $i, j = 1, 2, \ldots, n$. A matrix A is called nonnegative (positive) and denoted $A \geq 0$ $(A > 0)$ if $a_{ij} \geq 0$ $(a_{ij} > 0)$ for $i, j = 1, 2, \ldots, n$. A matrix $A \in \mathbb{R}^{n\times n}$ is called a Z-matrix if its off-diagonal entries are non-positive; an M-matrix if A is a Z-matrix and $A^{-1} \geq 0$; and an H-matrix if its comparison matrix $\langle A\rangle = (\langle a\rangle_{ij}) \in \mathbb{R}^{n\times n}$ is an M-matrix, where

$$\langle a\rangle_{ij} = \begin{cases} |a_{ij}| \text{ for } i = j, \\ -|a_{ij}| \text{ for } i \neq j, \end{cases} \quad i, j = 1, 2, \ldots, n.$$

Specially, an H-matrix with positive diagonal is called an H_+-matrix. If A is an M-matrix and B is a Z-matrix, then $A \leq B$ implies that B is an M-matrix. $|A|$ denotes the nonnegative matrix with entries $|a_{ij}|$, A^T denotes the transpose of the matrix A and $\rho(A)$ denotes the spectral radius of the matrix A.

For a given matrix $A \in \mathbb{R}^{n\times n}$, $A = M - N$ is called a splitting if M is nonsingular; a convergent splitting if $\rho(M^{-1}N) < 1$; an M-splitting if M is a nonsingular M-matrix and $N \geq 0$; an H-splitting if $\langle M\rangle - |N|$ is an M-matrix; and an H-compatible splitting if $\langle A\rangle = \langle M\rangle - |N|$. Note that an H-compatible splitting of an H-matrix is an H-splitting, but not vice versa. As is known, if $A = M - N$ is an M-splitting and A is a nonsingular M-matrix, then $\rho(M^{-1}N) < 1$.

3. Modulus-based matrix splitting iteration methods

In this section, we extend the modulus-based matrix splitting iteration method to solve the QCP (1.2).

Let $z - \Phi(z) = \frac{1}{\gamma}(|x| + x)$, $w = \frac{1}{\gamma}\Omega(|x| - x)$, and $g(z) = z - \Phi(z)$. Then $g(z)$ is invertible and $z = g^{-1}[\frac{1}{\gamma}(|x| + x)]$. Based on this fact, the following equivalence theorem is valid.

Theorem 3.1. *Let $A = M - N$ be a splitting of the matrix $A \in \mathbb{R}^{n\times n}$, $\gamma > 0$, and Ω be a positive diagonal matrix. For the QCP (1.2), the following statements hold true:*

(i) If (w, z) is a solution of the QCP (1.2), then $x = \frac{\gamma}{2}(z - \Omega^{-1}w - \Phi(z))$ satisfies the implicit fixed point equation

$$(\Omega + M)x = Nx + (\Omega - A)|x| - \gamma A\Phi[g^{-1}(\gamma^{-1}(|x| + x))] - \gamma\Psi[g^{-1}(\gamma^{-1}(|x| + x))] - \gamma q. \tag{3.1}$$

(ii) If x satisfies the implicit fixed point Eq. (3.1), then

$$z = \frac{1}{\gamma}(|x| + x) + \Phi(z), \text{ and } w = \frac{1}{\gamma}\Omega(|x| - x) \tag{3.2}$$

is a solution of the QCP (1.2).