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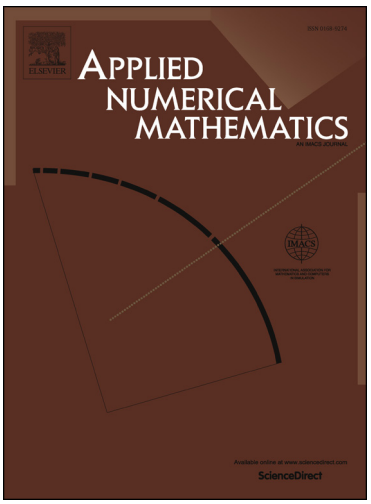

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The Spectral-Galerkin Approximation of Nonlinear Eigenvalue Problems

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Abstract

In this paper we present and analyze a polynomial spectral-Galerkin method for nonlinear elliptic eigenvalue problems of the form $-\mathrm{div}(A\nabla u)+Vu+f(u^2)u = \lambda u, \|u\|_{L^2} = 1$. We estimate errors of numerical eigenvalues and eigenfunctions. Spectral accuracy is proved under rectangular meshes and certain conditions of f. In addition, we establish optimal error estimation of eigenvalues in some hypothetical conditions. Then we propose a simple iteration scheme to solve the underlying an eigenvalue problem. Finally, we provide some numerical experiments to show the validity of the algorithm and the correctness of the theoretical results.

Keywords: Spectral-Galerkin approximation, error estimation, iteration algorithm, nonlinear eigenvalue problems

1 Introduction

Eigenvalue problems appear in many mathematical models for scientific and engineering applications, such as the calculation of the vibration modes of a mechanical structure in the framework of nonlinear elasticity, the Gross-Pitaevskii equation describing the steady states of Bose-Einstein condensates [14, 2], and the Hartree-Fock and Kohn-Sham equations used to calculate ground state electronic structures of molecular systems in quantum chemistry and materials science [6, 12, 16].

However, most of the existing analysis for eigenvalue approximations are concerned with linear eigenvalue problems [1], and there are relatively few results concerning approximation of nonlinear eigenvalue problems [18, 17, 7, 9, 10, 11, 13], and most of them are based on finite element methods with an exception in [7, 8] where an error estimate for Fourier spectral method to a periodic nonlinear eigenvalue problem is derived. To the best of our knowledge, there has no report on high order numerical methods for non-periodic nonlinear eigenvalue problems. Thus, the aim of this paper is to develop and analyze a spectral Galerkin method for a nonlinear elliptic eigenvalue problem. In particular, we shall extend the error estimates established in [7] for eigenvalues and eigenfunctions for periodic case

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