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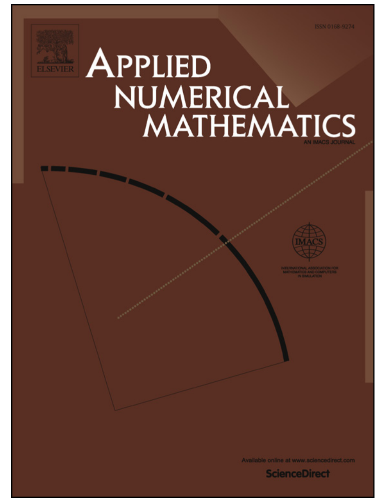

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The analysis of operator splitting methods for the Camassa–Holm equation

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Abstract

In this paper, the convergence analysis of operator splitting methods for the Camassa–Holm equation is provided. The analysis is built upon the regularity of the Camassa–Holm equation and the divided equations. It is proved that the solution of the Camassa–Holm equation satisfies the locally Lipschitz condition in H^1 and H^2 norm, which ensures the regularity of the numerical solution. Through the calculus of Lie derivatives, we show that the Lie–Trotter and Strang splitting converge with the expected rate under suitable assumptions. Numerical experiments are presented to illustrate the theoretical result.

Keywords: operator splitting, Lie–Trotter splitting, Strang splitting, convergence, Camassa–Holm equation

1. Introduction

The Camassa–Holm (CH) equation

$$u_t - u_{xxt} + 3uu_x = 2u_x u_{xx} + uu_{xxx},\ x \in \mathbb{R},\ t > 0, \tag{1}$$

proposed by Camassa and Holm [2], arises as a model for the propagation of unidirectional shallow water waves, with $u(x,t)$ representing the height of the fluid's free surface above a flat bottom. Such nonlinear dispersive partial differential equation is completely integrable and bi-Hamiltonian. It supports an infinite number of smooth waves (called solitons) and non-smooth solitary wave solutions (called peakons) and possesses an infinite number of conserved integrals. Because of these remarkable properties, it has attracted considerable concentration in recent years. A number of numerical methods have been proposed for the CH equation, such as pseudospectral method [18], finite difference

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