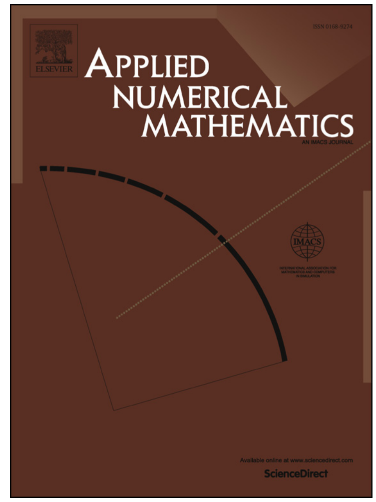

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Superconvergent IMEX Peer Methods

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Abstract

In this paper we will focus on numerical methods for differential equations with both stiff and nonstiff parts. This kind of problems can be treated efficiently by implicit-explicit (IMEX) methods and here we investigate a class of s-stage IMEX peer methods of order $p = s$ for variable and $p = s + 1$ for constant step sizes. They are combinations of s-stage superconvergent implicit and explicit peer methods. We construct methods of order $p = s+1$ for $s = 3, 4, 5$ where we compute the free parameters numerically to give good stability with respect to a general linear test problem frequently used in the literature. Numerical comparisons with two-step IMEX Runge-Kutta methods confirm the high potential of the new constructed superconvergent IMEX peer methods.

Keywords: IMEX peer methods, Superconvergence, Stability

1. Introduction

There are many problems and applications in engineering, physics, chemistry and other areas that lead to large systems of ordinary differential equations of the form:

$$y' = f(t, y) + g(t, y), \quad y(t_0) = y_0 \tag{1}$$

where $f : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ represents the stiff part of the equation and $g : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ represents the nonstiff part. Instead of using a single explicit or implicit method on the full problem, a more appropriate approach is to solve the nonstiff part by explicit methods due to their low cost per step. For the stiff part, explicit methods require small steps and for that reason implicit methods are used because their step size is not limited by stability requirements.

Combinations of implicit and explicit methods have been studied intensively in the literature. IMEX linear multistep methods have been proposed e.g. in [2, 13, 14, 19, 26] and IMEX Runge-Kutta methods in [1, 5, 16, 17]. The construction of high order IMEX Runge-Kutta methods is difficult due to the large number of order and coupling conditions between the explicit and the implicit part. IMEX multistep methods suffer from decreasing

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