Applied Numerical Mathematics 129 (2018) 26–38

Contents lists available at ScienceDirect

Applied Numerical Mathematics

www.elsevier.com/locate/apnum

A spectral method for triangular prism ☆

Jingliang Li [a,b], Heping Ma [b,*], Yonghui Qin [c], Shuaiyin Zhang [b]

[a] Zhangjiagang campus, Jiangsu University of Science and Technology, Zhangjiagang Jiangsu, 215600, China
[b] Department of Mathematics, College of Sciences, Shanghai University, Shanghai 200444, China
[c] College of Mathematics and Computing Science, Guilin University of Electronic Technology, Guilin, 541004, China

ARTICLE INFO

Article history:
Received 30 August 2017
Received in revised form 27 January 2018
Accepted 26 February 2018
Available online xxxx

Keywords:
One-to-one mapping
Integral singularity
Quasi-interpolation operator
Triangular prism spectral element method

ABSTRACT

In this paper, we study a spectral method for the triangular prism. We construct an approximation space in the "pole" condition in which the integral singularity is removed in a simple and effective way. We build a quasi-interpolation operator in the approximation space, and analyze its L^2-error. Based on the quasi-interpolation, a triangular prism spectral method for the elliptic modal problem is studied. Furthermore, we extend this triangular prism spectral method to a triangular prism spectral element method. For the elliptic modal problem, we present the spectral element scheme and analyze the convergence. At last, we do some experiments to test the effectiveness of the method.

1. Introduction

Spectral method, with its high accuracy, has become a popular method for simulations of fluid dynamics, atmospheric modeling and many other challenging problems [26,7,1,2,8,14,13,24]. Nevertheless, due to the global property of the basis functions, the classic spectral (element) methods are applied to the regular domain, i.e., square (right-angle polygon) in 2D and cube (right-angle polyhedron) in 3D, which restricts its further application to the domain with a complicated boundary. To break through this restriction, considerable efforts are devoted to the adaptability of spectral methods to the complicated domain problems. For the 3D complicated domain problems, due to the complexity, developing spectral methods for them is nontrivial. Some efforts are devoted to the tetrahedron [10,16,25]. Besides the tetrahedron, the prism is another important 3D domain. Some works are devoted to the methods for the prism. [12] studies a finite-difference time-domain method for a hexagonal prism problem; [32] and [9] study the finite volume methods for the right quadrangular prism and the straight triangular prism grid, respectively; [11,21,20,29,4] applied triangular prism finite element methods to solve various problems. However, these methods are low-order methods. So, developing a convenient high-order method for the columnar domain is valuable. Taking precision and flexibility into account, in this paper, we study a spectral method for the triangular prism, and further extend it to the triangular prism spectral element method (TPSEM).

Since the triangular prism is the tensor product of a triangle and a line segment, the spectral method for the triangular prism essentially depends on the spectral method on a triangle. Recently, [15] studies a nodal spectral element method on hybrid triangular and quadrilateral meshes. The mapped interpolation nodes on the triangle used in [15] are generated from the Legendre–Gauss–Lobatto (LGL) nodes on the reference square through the one-to-one mapping in [17]. It has more

☆ The work of the authors was supported in part by the National Natural Science Foundation of China (Grant No. 11571224, 11701119) and the open fund of Guangxi Key laboratory of hybrid computation and IC design analysis (HCIC201607).

* Corresponding author.
E-mail address: hpma@shu.edu.cn (H. Ma).

https://doi.org/10.1016/j.apnum.2018.02.009
0168-9274/

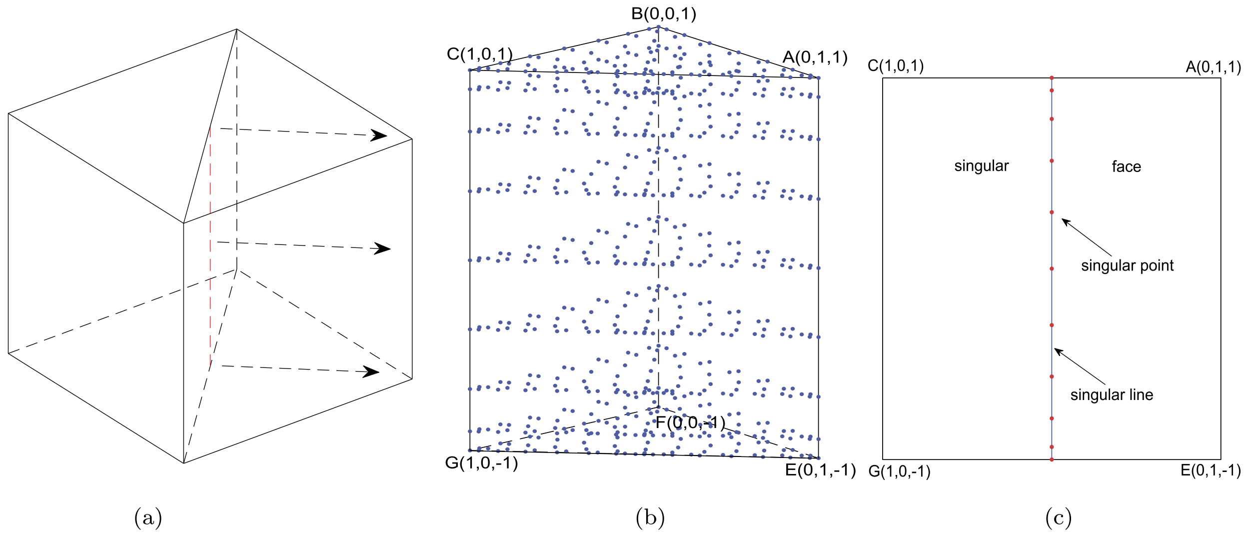

Fig. 2.1. (a) The mapping between the triangular prism and the reference cube. (b) the computational grid (mapped LGL points) on $\mathcal{P}$. (c) the diagram of the singular face, singular line and singular points on $\mathcal{P}$.

uniform distribution on the triangle than that of the mapped interpolation nodes produced by the Duffy mapping used in [5,6]. Another class of interpolation nodes on the triangle commonly used for the high order methods are related to optimal points for polynomial interpolation, such as the Feketes points, the electrostatic points and so on, see [3,28,30]. Compared with the mapped interpolation nodes, the distribution on the triangle of interpolation nodes in [3,28,30] is more uniform and convenient to extend to spectral element method. Based on this kind of interpolation nodes, [31,18,27,19] studied the nodal triangular spectral (element) methods. The effectiveness of these methods has been verified by the numerical examples therein. For easy to obtain the interpolation nodes on the triangle and enjoying the tensor-product property, the method in [15] is more convenient to be implemented, even if the approximation degree is large. In this paper, based on the method in [15], we explore a triangular prism spectral method (TPSM), and further extend it to TPSEM. This method possess the following advantages: at first, enjoying the tensor-product property enables it to assign matrices quickly; secondly, usage of the nodal basis enables it to handle the nonlinear or the variable coefficient problems; at last, the codes of this method is easy obtained through slightly revising that of the classic nodal basis spectral method, which makes this method convenient to realize.

This paper is organized as follows. In Section 2, we present the domain mapping from the reference cube to the standard triangular prism, and have a look at the integral singularity. In Section 3, we construct an approximation space in the “pole” condition, and introduce a quasi-interpolation operator. In Section 4, Based on the quasi-interpolation, we propose the triangular prism spectral method for the modal elliptic problem. In section 5, we extend this triangular prism spectral method to the TPSEM. We present the approximation scheme, analyze the convergence, and perform numerical tests. We make a conclusion in the last section.

2. Domain mapping

In this section, we present a one-to-one mapping between the reference cube and the standard triangular prism, and study the integral singularity brought by it.

Let (ξ,η,ζ) be the coordinate system on the reference cube $\mathcal{C}=\{(\xi,\eta,\zeta)|-1<\xi,\eta,\zeta<1\}$, and (x,y,z) be the coordinate system related to the triangular prism $\mathcal{P}=\{\boldsymbol{x}=(x,y,z)|0<x,y<1, y+x<1,-1<z<1\}$ (Fig. 2.1 (b)). The one-to-one mapping $\boldsymbol{F}:\mathcal{C}\to\mathcal{P}$ (see Fig. 2.1(a)), is given by

$$\begin{cases} x=\dfrac{(1+\xi)(3-\eta)}{8}, \\ y=\dfrac{(3-\xi)(1+\eta)}{8}, \qquad (\xi,\eta,\zeta)\in\overline{\mathcal{C}}. \\ z=\zeta. \end{cases} \tag{2.1}$$

Easy to see, transformation (2.1) pulls the edge $\xi=\eta=1$, $-1\le\zeta\le1$ of $\mathcal{C}$ to the line $x=y=\frac{1}{2}$, $-1\le z\le1$ of $\mathcal{P}$. By the direct calculation, we have