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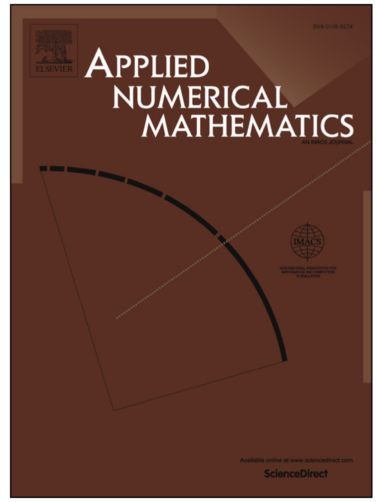

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The Taylor wavelets method for solving the initial and boundary value problems of Bratu-type equations

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Abstract

This paper presents an efficient numerical method for solving the initial and boundary value problems of the Bratu-type. In the proposed method, the Taylor wavelets are introduced, for the first time. An operational matrix of integration is derived and is utilized to reduce the Bratu-type initial and boundary value problems to a system of algebraic equations. Easy implementation, simple operations, and accurate solutions are the essential features of the proposed wavelets method. Illustrative examples are examined to demonstrate the performance and effectiveness of the developed approximation technique and a comparison is made with the existing results.

Keywords: Taylor wavelets, Bratu problems, operational matrix of integration, Numerical solution

1 Introduction

The Bratu's equations have drawn increasing attention and interest due to their important applications in science and engineering. A history and significance of the Bratu-type equations can be found in [1, 2].

Several chemical and physical processes are modeled using Bratu's equation [3]. To mention a few, this equation arises in the thermal reaction process in combustible, non-deformable material such as the solid in the fuel ignition model, the electro-spinning process for the production of ultrafine polymer fibers, the Chandrasekhar model of the expansion of the universe, chemical reactor theory, and nanotechnology [4–9].

A substantial amount of research work has been done for the study of Bratu's problem, including a method based on fixed-point iterations and Greens functions [9], an algorithm based on Chebyshev polynomial expansion [10], Laplace transform decomposition method [11], Adomian decomposition method [12], Laplace Adomain decomposition method [13], differential transformation method [14], variational iteration method [15, 16], Taylor's decomposition on two points [17], Spline method [7, 18], perturbation iteration method [19], Lie-group shooting method [20], the applications of Legendre wavelets method [21], Jacobi-Gauss collocation method [22], Sinc-Galerkin method [23], neural network, swarm intelligence and sequential quadratic programming [24], the successive differentiation method [25], finite difference method [26] and Block Nyström method [27].

Approximating by an orthogonal family of functions has found wide applications in various problems of the dynamical systems. The approach in using orthogonal functions is to convert the underlying differential equation into an integral equation through integration, approximating various signals involved in the equation by truncated orthogonal functions, and using the operational matrix of integration to eliminate the integral operations. This matrix can be uniquely determined based on the particular orthogonal functions. The Bernoulli polynomials and Taylor series are not based on orthogonal functions; nevertheless, they possess the operational matrix of integration [28, 29].

Wavelets are special kinds of oscillatory functions with compact support that provide the basis for many important spaces. They have been applied to a wide range of problems in science and

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