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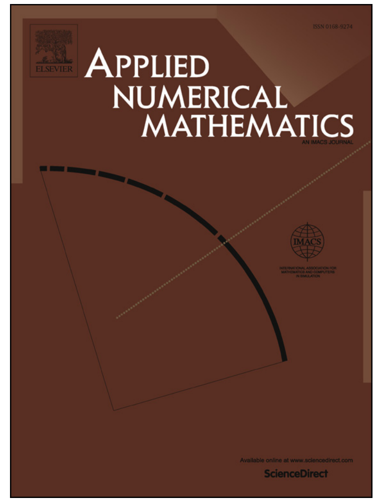

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DIAGONALIZED LEGENDRE SPECTRAL METHODS USING SOBOLEV ORTHOGONAL POLYNOMIALS FOR ELLIPTIC BOUNDARY VALUE PROBLEMS

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ABSTRACT. Fully diagonalized spectral methods using Sobolev orthogonal/biorthogonal basis functions are proposed for solving second order elliptic boundary value problems. We first construct the Fourier-like Sobolev polynomials which are mutually orthogonal (resp. bi-orthogonal) with respect to the bilinear form of the symmetric (resp. unsymmetric) elliptic Neumann boundary value problems. The exact and approximation solutions are then expanded in an infinite and truncated series in the Sobolev orthogonal polynomials, respectively. An identity is also established for the a posterior error estimate with a simple error indicator. Further, the Fourier-like Sobolev orthogonal polynomials and the corresponding Legendre spectral method are proposed in parallel for Dirichlet boundary value problems. Numerical experiments illustrate that our Legendre methods proposed are not only efficient for solving elliptic problems but also equally applicable to indefinite Helmoholtz equations and singular perturbation problems.

1. INTRODUCTION

Spectral methods for partial differential equations have gained more and more popularity during the past few decades. Fourier spectral methods make use of the eigenfunctions $e^{ikx}, k = 0, \pm 1, \pm 2, \ldots,$ of the Laplace operator which are orthogonal to each other with respect to the Sobolev inner product involving derivatives, thus the corresponding algebraic system is diagonal [2, 3, 12]. This fact together with the availability of the fast Fourier transform (FFT) makes the Fourier spectral method be an ideal approximation approach for differential equations with periodic boundary conditions. Meanwhile, for problems with non-periodic boundary conditions, Legendre spectral methods are of our greatest interests and also play a fundamental role in the study of spectral element methods.

Using Shen's polynomial basis, $\phi_k(x) = \frac{1}{\sqrt{4k-2}}\big(L_k(x) - L_{k-2}(x)\big) = \frac{\sqrt{4k-2}}{4k-4}(x^2-1)J^{1,1}_{k-2}(x), k = 2, 3, \ldots,$ the Legendre-Galerkin method for the Poisson equation $-u''(x) = f(x)$ on $(-1, 1)$ with homogenous Dirichlet boundaries $u(\pm 1) = 0$ leads to an algebraic system with the identity matrix [11]. Actually, the basis functions are Sobolev orthonormal polynomials in $H^1_0(-1, 1)$ with respect to the equivalent inner product $\langle u, v\rangle = (u', v') := \int_{-1}^1 u'(x)v'(x)dx$. Thus the exact and numerical

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