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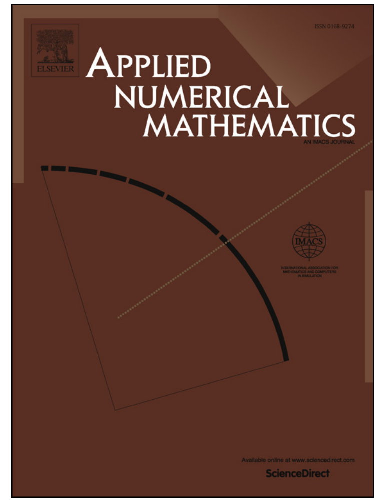

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Marzieh Dehghani-Madiseh, Milan Hladík

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Efficient Approaches for Enclosing the United Solution Set of the Interval Generalized Sylvester Matrix Equations

Marzieh Dehghani-Madiseh[a], Milan Hladík[b,*]

[a]Department of Mathematics, Faculty of Mathematical Sciences and Computer, Shahid Chamran University of Ahvaz, Ahvaz, Iran.

[b]Charles University, Faculty of Mathematics and Physics, Department of Applied Mathematics, Malostranské nám. 25, 11800, Prague, Czech Republic.

Abstract

We investigate the interval generalized Sylvester matrix equation $\mathbf{A}X\mathbf{B}+\mathbf{C}X\mathbf{D}=\mathbf{F}$. We propose a necessary condition for its solutions, and also a sufficient condition for boundedness of the whole solution set. The main effort is performed to develop techniques for computing outer estimations of the so-called united solution set of this interval system. First, we propose a modified variant of the Krawczyk operator, reducing significantly computational complexity, compared to the Kronecker product form. We then propose an iterative technique for enclosing the solution set. These approaches are based on spectral decompositions of the midpoints of $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ and in both of them we suppose that the midpoints of $\mathbf{A}$ and $\mathbf{C}$ are simultaneously diagonalizable as well as for the midpoints of the matrices $\mathbf{B}$ and $\mathbf{D}$. Numerical experiments are given to illustrate the performance of the proposed methods.

Keywords: Interval arithmetic, Generalized Sylvester matrix equation, Krawczyk operator, Preconditioning.
2000 MSC: 65G30, 15A24

1. Introduction

Consider the implicit differential equation

$$g(\dot{x},x)=0. \tag{1}$$

As written in [1], to obtain a numerical solution of (1) using the block multistep method suppose one has available quantities $(\dot{x}_{j,n-p},x_{j,n-p})$ that approximate $\dot{x}(t)$ and $x(t)$ at past times $t_{j,n-p}=t_{n-p-1}+hc_j$, where h is the time step, $j=1,\ldots,v$ and $p=1,\ldots,k$ (usually the numbers c_j satisfy $0\leq c_j\leq 1$). To advance the method from time t_{n-1} to time t_n, the quantities $x_{j,n},\dot{x}_{j,n}$ at stage l of the iterations must be satisfied in the following conditions

$$g(\dot{x}^{(l)}_{j,n},x^{(l)}_{j,n})+[\partial g/\partial\dot{x}]\delta\dot{x}^{(l)}_{j,n}+[\partial g/\partial x]\delta x^{(l)}_{j,n}=0,\qquad j=1,\ldots,v, \tag{2}$$

*Corresponding author
Email addresses: marzieh.dehghani66@gmail.com (Marzieh Dehghani-Madiseh), milan.hladik@matfyz.cz (Milan Hladík)