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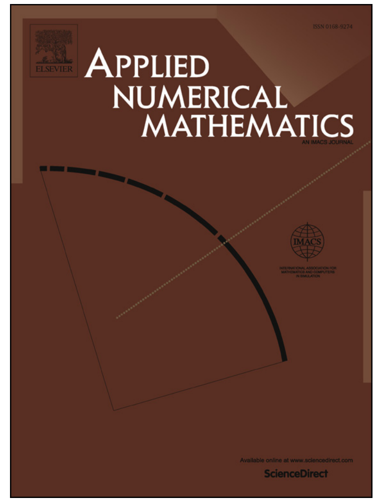

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Shape optimization for Stokes flows using sensitivity analysis and finite element method

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SUMMARY

In the context of structural optimization in fluid mechanics we propose a numerical method based on a combination of the classical shape derivative and Hadamard's boundary variation method. Our approach regards the viscous flows governed by Stokes equations with the objective function of energy dissipation and a constrained volume. The shape derivative is computed by Lagrange's approach via the solutions of Stokes and adjoint systems. The programs are written in FreeFem++ using the Finite Element method.

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Keywords: Shape optimization, Sensitivity analysis, Shape derivative and Stokes equations.

1. INTRODUCTION

Shape optimization in fluid mechanics has received a large amount of attention from both engineers and mathematicians since the pioneering works of Pironneau [Pir73, Pir74]. It has brought forth many interesting applications: the optimal shape design (OSD) of airplane wing which induces minimal drag, i.e. minimal reaction from the surrounding fluid in aeronautic industry, see [MP04, POTTP06]; the optimal control approaches to shape optimization of aorto-coronaric bypass anastomoses, applied in [QR03, AQR06a, AQR06b]; the problem of optimal swimming of microorganisms at low Reynolds number, considered in [ADH11] (more specifically, the authors are in search for the shape of such microorganism that allows to reach a prescribed displacement while undergoing minimal stress from the surrounding fluid, the shape of the microorganisms is parametrized by means of a few physical parameters, and the sensitivity of the stress exerted by the fluid on the microorganisms with respect to perturbations on these parameters is computed). In addition, an interesting application of the topology optimization method for fluid flow can be found in [BP03] for minimum power dissipation Stokes flow problems in two dimensions. Since then, alternative parameterizations have been suggested for the Stokes flow problem [GP06] and the Navier-Stokes flow problem [Evg06]. Topology optimization for Stokes flow has been extended to large-scale problems [APGHS08] and applied to fluid flows with regions of Darcy and Stokes flow [WKB07] and with low-to-moderate Reynolds numbers [OOB06, DMZ08a, ZL08]. For more works and many other applications of shape or topology optimization concepts in fluid mechanics, we refer to [HN96, BS03, Gun03, MP04, MP10] and the references therein.
In this article our aim is to construct a numerical scheme for the problem of shape optimization in fluid mechanics. Let $\Omega \in \mathbb{R}^d (d = 2 \,\mathrm{or}\, 3)$ be a bounded open set occupied by the fluid governed by

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