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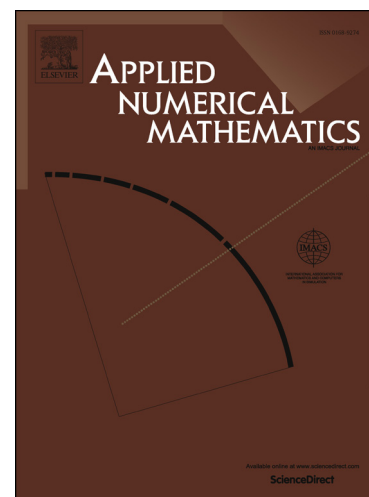
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# Mixed recurrence equations and interlacing properties for zeros of sequences of classical $q$ -orthogonal polynomials

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## Abstract

Using the  $q$ -version of Zeilberger's algorithm, we provide a procedure to find mixed recurrence equations satisfied by classical  $q$ -orthogonal polynomials with shifted parameters. These equations are used to investigate interlacing properties of zeros of sequences of  $q$ -orthogonal polynomials. In the cases where zeros do not interlace, we give some numerical examples to illustrate this.

*Keywords:* classical  $q$ -orthogonal polynomials, Mixed recurrence equations, Interlacing of zeros  
*2010 MSC:* 33C05, 33C45, 33F10, 33D15

## 1. Introduction

Let  $0 < q < 1$ . The classical  $q$ -orthogonal polynomials were introduced by Hahn [8] and can be written in terms of basic hypergeometric series, as introduced by Heine [9] in 1847. These polynomials are associated especially to quantum groups (cf. [16, 18, 19]), as introduced in [4, 26]. We list the systems of monic  $q$ -orthogonal polynomials considered in this paper (cf. [15]).

### 1. Askey-Wilson polynomials

$$\tilde{p}_n(x; a, b, c, d|q) = \frac{(ab, ac, ad; q)_n}{(2a)^n (abcdq^{n-1}; q)_n} {}_4\phi_3 \left( \begin{matrix} q^{-n}, abcdq^{n-1}, ae^{i\theta}, ae^{-i\theta} \\ ab, ac, ad \end{matrix} \middle| q; q \right), \quad x = \cos \theta, \quad (1)$$

with  $a, b, c, d$  either real, or they occur in complex conjugate pairs, and  $\max(|a|, |b|, |c|, |d|) < 1$ ,  $x \in (-1, 1)$ ;

### 2. $q$ -Racah polynomials

$$\tilde{R}_n(\mu(x); \alpha, \beta, \gamma, \delta|q) = \frac{(\alpha q, \beta \delta q, \gamma q; q)_n}{(\alpha \beta q^{n+1}; q)_n} {}_4\phi_3 \left( \begin{matrix} q^{-n}, \alpha \beta q^{n+1}, q^{-x}, \gamma \delta q^{x+1} \\ \alpha q, \beta \delta q, \gamma q \end{matrix} \middle| q; q \right), \quad \mu(x) = q^{-x} + \gamma \delta q^{x+1}, \quad (2)$$

$n \in \{0, 1, \dots, N\}$ ,  $\alpha q = q^{-N}$  or  $\beta \delta q = q^{-N}$  or  $\gamma q = q^{-N}$ ,  $N$  a nonnegative integer;

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