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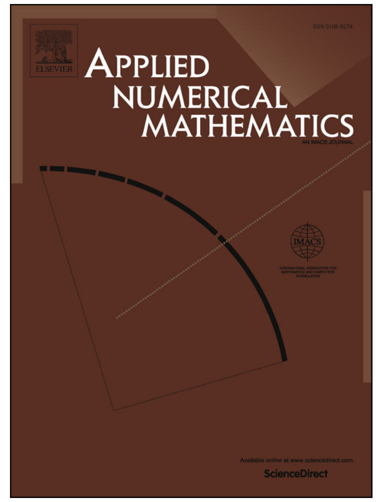

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Mixed recurrence equations and interlacing properties for zeros of sequences of classical q-orthogonal polynomials

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Abstract

Using the q-version of Zeilberger's algorithm, we provide a procedure to find mixed recurrence equations satisfied by classical q-orthogonal polynomials with shifted parameters. These equations are used to investigate interlacing properties of zeros of sequences of q-orthogonal polynomials. In the cases where zeros do not interlace, we give some numerical examples to illustrate this.

Keywords: classical q-orthogonal polynomials, Mixed recurrence equations, Interlacing of zeros
2010 MSC: 33C05, 33C45, 33F10, 33D15

1. Introduction

Let $0 < q < 1$. The classical q-orthogonal polynomials were introduced by Hahn [8] and can be written in terms of basic hypergeometric series, as introduced by Heine [9] in 1847. These polynomials are associated especially to quantum groups (cf. [16, 18, 19]), as introduced in [4, 26]. We list the systems of monic q-orthogonal polynomials considered in this paper (cf. [15]).

1. Askey-Wilson polynomials

$$\tilde{p}_n(x;a,b,c,d|q) = \frac{(ab,ac,ad;q)_n}{(2a)^n(abcdq^{n-1};q)_n}\,{}_4\phi_3\left(\begin{matrix} q^{-n}, abcdq^{n-1}, ae^{i\theta}, ae^{-i\theta} \\ ab, ac, ad \end{matrix}\,\middle|\, q;q\right),\ x=\cos\theta, \quad (1)$$

with a,b,c,d either real, or they occur in complex conjugate pairs, and $\max(|a|,|b|,|c|,|d|) < 1$, $x \in (-1,1)$;

2. q-Racah polynomials

$$\tilde{R}_n(\mu(x);\alpha,\beta,\gamma,\delta|q) = \frac{(\alpha q,\beta\delta q,\gamma q;q)_n}{(\alpha\beta q^{n+1};q)_n}\,{}_4\phi_3\left(\begin{matrix} q^{-n}, \alpha\beta q^{n+1}, q^{-x}, \gamma\delta q^{x+1} \\ \alpha q, \beta\delta q, \gamma q \end{matrix}\,\middle|\, q;q\right), \mu(x) = q^{-x}+\gamma\delta q^{x+1}, \quad (2)$$

$n \in \{0,1,\ldots,N\}$, $\alpha q = q^{-N}$ or $\beta\delta q = q^{-N}$ or $\gamma q = q^{-N}$, N a nonnegative integer;

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