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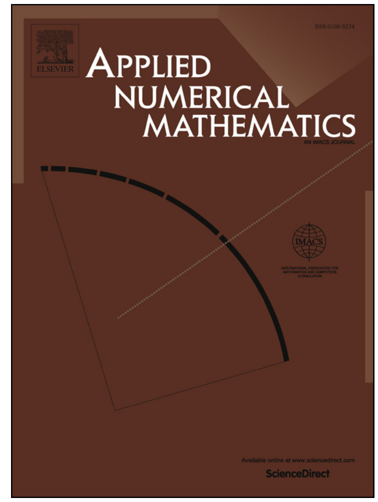

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PII: S0168-9274(17)30240-4
DOI: https://doi.org/10.1016/j.apnum.2017.11.005
Reference: APNUM 3281

To appear in: *Applied Numerical Mathematics*

Received date: 20 October 2016
Revised date: 29 August 2017
Accepted date: 18 November 2017

Please cite this article in press as: E. Ideon, P. Oja, Quadratic/linear rational spline collocation for linear boundary value problems, *Appl. Numer. Math.* (2018), https://doi.org/10.1016/j.apnum.2017.11.005

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Quadratic/linear rational spline collocation for linear boundary value problems

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Abstract

We investigate the collocation method with quadratic/linear rational spline S of smoothness class C^2 for the numerical solution of two-point boundary value problems if the solution y (or $-y$) of the boundary value problem is a strictly convex function. We show that on the uniform mesh it holds $\|S - y\|_\infty = O(h^2)$. Established bound of error gives a dependence on the solution of the boundary value problem and its coefficient functions. We prove also convergence rates $\|S' - y'\|_\infty = O(h^2)$ and $\|S'' - y''\|_\infty = O(h^2)$. Numerical examples support the obtained theoretical results.

Keywords: Boundary value problems, collocation, rational spline, convergence
2010 MSC: 65D05, 65D07, 65L10

1. Introduction

We studied in [6] the linear/linear rational spline collocation for linear boundary value problems. The existence of solution was established and convergence rate $O(h^2)$ found out with the main term of error estimate. The convergence $O(h^2)$ with main terms of error estimate are known also at quadratic spline and cubic spline collocation [7, 11]. All these three are different and none of them has advantage in comparison with others. The research practice shows that the convergence rate is directly influenced by the interpolation properties of splines under study. The interpolation with polynomial splines is a classical topic. The interpolation with linear/linear and quadratic/linear rational splines is studied as well [4, 5] and asymptotic expansions of the interpolant, its superconvergence properties are known. Consequently, it is very natural to pose the question about similar problems at quadratic/linear rational spline collocation. It occurs that, comparing with linear/linear rational spline case, quite different ideas should be used. In both cases the proof of existence of the solution is based on Bohl-Brouwer fixed point theorem as a main tool in nonlinear analysis. While the linear/linear rational spline case uses spline values, at the quadratic/linear rational spline collocation we use spline values and derivative values in knots. It is interesting to mention that the operator in Bohl-Brouwer theorem is obtained from basic equations in a way which appears in Gauss-Seidel method.

We considered only the simplest boundary conditions. More general conditions at quadratic and cubic spline collocation are treated in [8, 10] and they create considerable technical difficulties. Our proofs in this paper are by themselves technically complicated and we confine ourselves to the simple boundary conditions.

A short overview about similar problems and researches of other authors is given in [6]. We refer the reader only to the books [2, 3, 13] for thorough treatment and comprehensive bibliography.

The quadratic/linear rational spline collocation is reasonable only if the solution of initial problem is strictly convex or strictly concave because such geometrical property is proper to splines of such kinds. However, this restricts the use of this method. But in general case, an adaptive strategy could be used combining quadratic/linear rational splines on convex region with cubic splines on intervals of nonconvexity. This problem needs to be investigated in future and clearly should be based on the results of present paper. Let us mention that the theory of adaptive interpolation could be find in [9, 12].

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