



# Alliances in graphs: Parameters, properties and applications—A survey

Kahina Ouazine, Hachem Slimani\*, Abdelkamel Tari

LIMED Laboratory, Computer Science Department, University of Bejaia, 06000 Bejaia, Algeria

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## Abstract

In practice, an alliance can be a bond or connection between individuals, families, states, or entities, etc. Formally, a non-empty set  $S$  of vertices of a graph  $G$  is a defensive  $k$ -alliance (resp. an offensive  $k$ -alliance) if every vertex of  $S$  (resp. the boundary of  $S$ ) has at least  $k$  more neighbors inside of  $S$  than it has outside of  $S$ . A powerful  $k$ -alliance is both defensive  $k$ -alliance and offensive  $(k+2)$ -alliance. During the last decade there has been a remarkable development on these three kinds of alliances. Due to their variety of applications, the alliances in its broad sense have received a special attention from many scientists and researchers. There have been applications of alliances in several areas such as bioinformatics, distributed computing, web communities, social networks, data clustering, business, etc. Several  $k$ -alliance numbers have been defined and a huge number of theoretical (algorithmic and computational) results are obtained for various graph classes. In this paper, we present a survey which covers a number of practical applications of alliances and the vast mathematical properties of the three types of  $k$ -alliances by giving a special attention to the study of the associated  $k$ -alliance (partition) numbers for different graph classes.

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**Keywords:** Defensive (offensive powerful)  $k$ -alliance; Boundary  $k$ -alliance; Partitioning of graphs;  $k$ -alliance (partition) number; Graph class

## Contents

1. Introduction .....	2
1.1. Historical view and applications of alliances .....	2
1.2. Terminology and definitions .....	4
2. Defensive $k$ -alliances in graphs .....	5
2.1. Study of defensive $k$ -alliance numbers for various graphs classes .....	6
2.1.1. General graphs .....	6
2.1.2. Tree graphs .....	7
2.1.3. Planar graphs .....	8
2.1.4. Complete graphs .....	9
2.1.5. Bipartite graphs and complete bipartite graphs .....	9

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\* Corresponding author. Fax: +213 34 813709.

E-mail addresses: [ouazine.kahina@gmail.com](mailto:ouazine.kahina@gmail.com) (K. Ouazine), [haslimani@gmail.com](mailto:haslimani@gmail.com) (H. Slimani), [tarikamel59@gmail.com](mailto:tarikamel59@gmail.com) (A. Tari).

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2.1.6.	Regular graphs .....	10
2.1.7.	Cycle graphs .....	10
2.1.8.	Path graphs .....	10
2.1.9.	Line graphs .....	11
2.1.10.	Cartesian product graphs .....	11
2.2.	Study of defensive $k$ -alliance partition numbers for some graph classes .....	12
2.2.1.	General graphs .....	16
2.2.2.	Tree graphs .....	16
2.2.3.	Regular graphs .....	17
2.2.4.	Cartesian product graphs .....	17
2.2.5.	Partitioning a graph into boundary defensive $k$ -alliances .....	17
3.	Offensive $k$ -alliances in graphs .....	19
3.1.	Study of offensive $k$ -alliance numbers for various graph classes .....	19
3.1.1.	General graphs .....	19
3.1.2.	Tree graphs .....	21
3.1.3.	Planar graphs .....	21
3.1.4.	Complete graphs .....	22
3.1.5.	Bipartite graphs and complete bipartite graphs .....	22
3.1.6.	Regular graphs .....	22
3.1.7.	Cycle graphs .....	23
3.1.8.	Path graphs .....	23
3.1.9.	Line graphs .....	23
3.1.10.	Cartesian product graphs .....	23
3.2.	Study of offensive $k$ -alliance partition numbers .....	27
3.2.1.	General graphs .....	27
3.2.2.	Cartesian product graphs .....	28
3.2.3.	Circulant graphs — $CR(n, 2)$ .....	28
3.2.4.	Relations between $\psi_k^{go}(G)$ and $k$ .....	29
3.2.5.	Partition number and chromatic number .....	29
4.	Powerful $k$ -alliances in graphs .....	29
4.1.	Study of powerful $k$ -alliance numbers for various graph classes .....	30
4.1.1.	General graphs .....	30
4.1.2.	Tree graphs .....	31
4.1.3.	Planar graphs .....	32
4.1.4.	Complete graphs .....	32
4.1.5.	Complete bipartite graphs .....	32
4.1.6.	Regular graphs .....	33
4.1.7.	Cycle graphs .....	33
4.1.8.	Path graphs .....	33
4.1.9.	Cartesian product graphs .....	33
4.2.	Study of powerful $k$ -alliance partition numbers .....	35
4.2.1.	General graphs .....	35
4.2.2.	Cartesian product graphs .....	35
4.2.3.	Partitioning a graph into boundary powerful $k$ -alliances .....	35
5.	Conclusion and discussion .....	37
	Acknowledgments .....	37
	Appendix .....	37
	References .....	38

## 1. Introduction

### 1.1. Historical view and applications of alliances

The word alliance can be defined as a union or association formed for mutual benefit, for example it can be: a formal agreement or treaty between two or more nations to cooperate for specific purposes, a merging of efforts or interests by persons, families, states, or organizations. The study of alliances in graphs is first investigated by Kristiansen et al. [1] by defining different types of alliances that have been extensively studied in the last decade. These types of alliances are called defensive alliances [2,3], offensive alliances [4,5] and dual or powerful alliances [6,7]. A generalization of these alliances called  $k$ -alliances (or  $r$ -alliances) introduced by Shafique and Dutton [8,9] have received a special attention in recent years. In this setting, there have been definitions of many and various parameters which have been

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