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On total edge irregularity strength of centralized uniform theta graphs

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Abstract

Let $G = (V, E)$ be a simple connected and undirected graph. Let $f : V \cup E \to \{1, 2, \ldots, k\}$ be a total labeling of G. The weight of an edge uv is defined by $w_f(uv) = f(u) + f(v) + f(uv)$. The labeling f is called an edge irregular total k-labeling if $w_f(uv) \neq w_f(u'v')$ for any two distinct edges uv, $u'v'$. If G admits such a labeling, then the minimum k is called the total edge irregularity strength of G. In this paper we determine the total edge irregularity strength of centralized uniform theta graphs.

Keywords: Irregular total labeling; Total edge irregularity strength; Uniform theta graphs

1. Introduction

Let $G = (V, E)$ be a simple, connected and undirected graph. Motivated by [1], Bača et al. defined the so-called *edge irregular total k-labeling* on graph $G = (V, E)$, as a total k-labeling $f : V \cup E \to \{1, 2, \ldots, k\}$ such that the weights of all edges are distinct [2]. The weight of edge uv under k-labeling f, denoted by $wt_f(uv)$, is calculated by summing the label of u, the label of v and the label of uv, i.e. $wt_f(uv) = f(u) + f(uv) + f(v)$. The minimum k such that G admits an edge irregular total k-labeling is called *the total edge irregularity strength* of G, denoted by $tes(G)$ [2].

Bača et al. in [2] gave a result on the lower bound of total edge irregularity strength of any graph G with maximum degree $\Delta(G)$ as follows

$$tes(G) \geq max \left\{ \left\lceil \frac{|E| + 2}{3} \right\rceil, \left\lceil \frac{\Delta(G) + 1}{2} \right\rceil \right\}. \quad (1)$$

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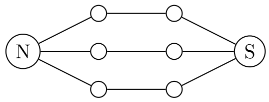

Fig. 1. Uniform theta graph $\theta(3; 2)$.

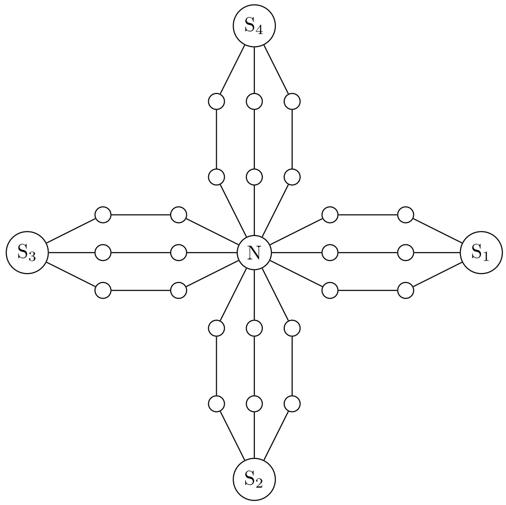

Fig. 2. Centralized uniform theta graph $\theta^*(3; 2; 4)$.

Motivated by this result, Ivančo and Jendrol in [3] determined the total edge irregularity strength of trees. Moreover, they posed a conjecture stating that for any graph G different from K_5, the total edge irregularity strength of G is precisely the lower bound of (1). Recently, it was proven, the conjecture is true for subdivision of star graphs (in [4,5]) and for series parallel graphs (in [6]).

In [7], Rajan et al. defined a *generalized theta graph* $\theta(s_1, s_2, \dots, s_n)$ consisting a pair of end vertices joined by n internal disjoint paths of length at least two, where s_i denote the number of internal vertices in the ith path. The end vertices are called north pole (N) and south pole (S), respectively. A generalized theta graph is called *uniform theta graph* if all paths connecting the two poles have the same number of internal vertices. We denote by $\theta(n; m)$ for the uniform theta graph with $n \geq 3$ paths connecting the poles and $m \geq 1$ internal vertices in each path. Fig. 1 shows uniform theta graph $\theta(3; 2)$ with 3 internal disjoint paths and 2 internal vertices.

In [6], Rajasingh and Arockiamary constructed a graph by composing a series of uniform theta graphs, that is, by merging the south pole of one uniform theta graph onto the north pole of another uniform theta graph. They managed to obtain the total edge irregularity strength of the constructed graph. While in [4], Siddiqui constructed a subdivision of star, denoted by S_n^m, by merging one of the end vertices of some paths. However, the total edge irregularity strength of S_n^m for $m \geq 9$ and $n \geq 3$ was becoming an open problem until Hinding et al. determined it in [5].

Motivated by [4–6], we construct a centralized uniform theta graph by collecting some uniform theta graphs of the same type and merging one of their poles. We denote by $\theta^*(n; m; p)$ for centralized uniform theta graphs, constructed from $p \geq 3$ uniform theta graphs $\theta(n; m)$. Fig. 2 shows the centralized uniform theta graph $\theta^*(3; 2; 4)$ which is constructed from 4 uniform theta graphs $\theta(3; 2)$.

We name the vertices of the uniform theta graph $\theta^*(n; m; p)$ in this way: we denote the merged pole by c_0, the jth internal vertex of ith path from the lth uniform theta graph $\theta(n; m)$ by $x_{i,j,l}$, and the unmerged poles of lth uniform theta graph $\theta(n; m)$ by c_l. Fig. 3 shows the centralized uniform theta graph $\theta^*(n; m; p)$ with such notation.

From the above notation, for the centralized uniform theta graph $\theta^*(n; m; p) = (V, E)$, we have $V = \{c_0\} \cup \{c_l | 1 \leq l \leq p\} \cup \{x_{i,j,l} | 1 \leq i \leq n, 1 \leq j \leq m, 1 \leq l \leq p\}$ and $E = \{c_0 x_{i,1,l} | 1 \leq i \leq n, 1 \leq l \leq p\} \cup \{x_{i,m,l} c_l | 1 \leq i \leq n, 1 \leq l \leq p\} \cup \{x_{i,j-1,l} x_{i,j,l} | 1 \leq i \leq n, 2 \leq j \leq m, 1 \leq l \leq p\}$. Hence, $|V| = (nm + 1)p + 1$, $|E| = n(m + 1)p$, and $\Delta(\theta^*(n; m; p)) = np$.