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Structure connectivity of hypercubes

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Abstract

The connectivity of a graph is an important measurement for the fault-tolerance of the network. To provide more accurate measures for the fault-tolerance of networks than the connectivity, some generalizations of connectivity have been introduced. Let H be a connected subgraph of a graph G. A set F of a connected subgraphs of G is called a subgraph cut of G if $G - F$ is either disconnected or trivial. If further, each member of F is isomorphic to H, then F is called an H-structure cut of G. The H-structure connectivity $\kappa(G; H)$ of G is the minimum cardinality of an H-structure cut of G. In this paper we determine $\kappa(Q_n; H)$ or its upper bound where Q_n is the n-dimensional hypercube with $n \geq 4$ and H is either Q_m with $m \leq n-2$ or even cycle C_l with $l \leq 2^n$.

Keywords: Structure connectivity; Cycle; Hypercube

1. Introduction

In the design of an interconnection system, one important consideration is that the network should be least vulnerable to any disruption. The ability of a system to continue operations correctly in the presence of failures in one or many of its components is known as fault tolerance. The connectivity is one of the essential parameters to evaluate the fault tolerance of a network. To make an overall evaluation of an interconnection network with failures, some other measures related to connectivity have been introduced and studied in recent years [1–3].

Lin et al. [4] introduced a new kind of connectivity called structure connectivity. According to them, a set F of connected subgraphs of G is a subgraph cut of G if $G - V(F)$ is disconnected or trivial graph. Let H be a connected subgraph of G. Then F is an H-structure-cut if F is subgraph cut, and every element in F is isomorphic to H. They defined the H-structure connectivity of G denoted by $\kappa(G; H)$, to be the minimum cardinality of all H-structure-cuts of G. This study was motivated by the trend that networks and subnetworks of large scales are increasingly made into chips, and it is becoming more and more feasible to consider the fault status of a structure, rather than individual nodes. They obtained $\kappa(Q_n; H)$ for each $H \in (K_1, K_{1,1}, K_{1,2}, K_{1,3}, C_4)$.

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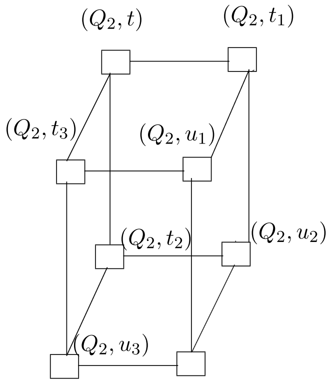

Fig. 1. $Q_5 = Q_2 \square Q_3$.

In this paper, we determine $\kappa(Q_n; H)$ or its upper bound where Q_n is the n-dimensional hypercube with $n \geq 4$ and H is Q_m where $m \leq n-2$ and an even cycle C_l with $l \leq 2^n$.

For a positive integer n, we denote the hypercube graph Q_n whose vertex set i.e. $V(Q_n) = \{u = u_1 u_2 \ldots u_n : u_k \in \{0, 1\}, 1 \leq k \leq n\}$ consisting of binary strings of length n. Two strings are adjacent if they differ in exactly one-bit position. Q_n has many attractive properties, such as being bipartite, n-regular, n-connected, edge-bipancyclic. Due to these and many more attractive topological properties, hypercube has been one of the most fundamental interconnection networks.

We decompose $Q_n = Q_m \square Q_{n-m}$. Now, for any $t \in V(Q_{n-m})$ we denote by (Q_m, t) the subgraph of Q_n induced by the vertices whose last $n-m$ components form the tuple t. It is easy to observe that (Q_m, t) is isomorphic to Q_m.

For undefined terminology and notations see [4,5].

2. Structure connectivity of hypercubes

Theorem 2.1. *Let $n \geq 4$ be an integer. Then for each $1 \leq m \leq (n-2)$, $\kappa(Q_n; Q_m) = n-m$ and $\kappa(Q_n; C_{2^m}) \leq n-m$.*

Proof. Let $Q_n = Q_m \square Q_{n-m}$. As Q_{n-m} is $(n-m)$-regular and $(n-m)$-connected, every vertex in $V(Q_{n-m})$ is adjacent to exactly $n-m$ number of vertices. Let $t \in V(Q_{n-m})$ be adjacent to $t_1, t_2, \ldots, t_{(n-m)} \in V(Q_{n-m})$. Hence induced subgraph (Q_m, t) of Q_n is adjacent to exactly $(n-m)$ subcubes namely $(Q_m, t_1), (Q_m, t_2), \ldots, (Q_m, t_{(n-m)})$ (see Fig. 1 for an illustration). Clearly, removal of $\bigcup_{i=1}^{n-m}(Q_m, t_i)$ disconnects Q_n. Thus, $\kappa(Q_n; Q_m) \leq n-m$. If $\kappa(Q_n; Q_m) < n-m$, then without loss of generality one can assume that removal of $\bigcup_{i=2}^{n-m}(Q_m, t_i)$ disconnects Q_n. Hence the removal of $\bigcup_{i=2}^{n-m} t_i$ will disconnect Q_{n-m}, a contradiction to Q_{n-m} is $(n-m)$-connected. Hence $\kappa(Q_n; Q_m) = n-m$.

Since Q_{n-m} is Hamiltonian, it contains a spanning cycle of length 2^{n-m}. Removal of isomorphic Hamiltonian cycles of all subcubes $(Q_m, t_1), (Q_m, t_2), \ldots, (Q_m, t_{(n-m)})$ disconnect Q_n. Hence $\kappa(Q_n; C_{2^m}) \leq n-m$. $\square$

Proposition 2.2. *Let $n \geq 5$ be an integer. Then for any integer m with $2 \leq m \leq n-2$ and for any even integer k with $2^m < k < 2^{m+1}$, $\kappa(Q_n; C_k) \leq n-m$.*

Proof. Let $Q_n = Q_m \square Q_{n-m}$ $(2 \leq m \leq n-2)$. Let $t \in V(Q_{n-m})$ be adjacent to $n-m$ number of vertices say $t_1, t_2, \ldots, t_{(n-m)}$. It is well known that hypercube is a bipartite graph so, t_i is not adjacent to t_j for all $i \neq j$ $(1 \leq i, j \leq n-m)$. Also, every pair of adjacent edges lies in exactly one 4-cycle [6]. Hence we name vertices say $u_1, u_2, \ldots, u_{(n-m)}$ from $V(Q_{n-m})$ such that t_i is adjacent to say u_i for all $1 \leq i \leq n-m$ (see Fig. 1 for an illustration).

In the case when $n-m = 2$, we take vertices of $Q_{n-m} = Q_2$ as t_1, u_1, t_2, u_2. Hence the 4−cycle in Q_2 we take as $t_1 u_1 t_2 u_2 t_1$. See Fig. 2.

The construction of cycles of length $2^m + l$ for every even integer l $(2 \leq l \leq 2^m - 2)$ proceeds as follows. We start by choosing an arbitrary Hamiltonian cycle say C^i of (Q_m, t_i) for every i $(1 \leq i \leq n-m)$. Then we choose any edge