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Hybrid methods for a finite family of G-nonexpansive mappings in Hilbert spaces endowed with graphs

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Abstract

In this paper, we prove a strong convergence theorem for two different hybrid methods by using CQ method for a finite family of G-nonexpansive mappings in a Hilbert space. We give an example and numerical results for supporting our main results and compare the rate of convergence of the two iterative methods.
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1. Introduction

Let H be a Hilbert space with the inner product $\langle ., . \rangle$, norm $\|.\|$ and C be a nonempty subset of H. A nonlinear mapping $T : C \to C$ is said to be

1. *contraction* if there exists $\alpha \in (0, 1)$ such that $\|Tx - Ty\| \le \alpha\|x - y\|$ for all $x, y \in C$;
2. *nonexpansive* if $\|Tx - Ty\| \le \|x - y\|$ for all $x, y \in C$.

The fixed point set of T is denoted by $F(T)$, that is, $F(T) = \{x \in C : x = Tx\}$.

Since 1922, fixed point theorems and the existence of fixed points of a single-valued nonlinear mapping have been intensively studied and considered by many authors (see, for examples [1–7]).

In 1953, Mann [8] introduced the famous iteration procedure as follows:

$$x_1 \in C, \quad x_{n+1} = \alpha_n x_n + (1 - \alpha_n)Tx_n, \ \forall n \in \mathbb{N}$$

where $\{\alpha_n\} \subset [0, 1]$ and $\mathbb{N}$ the set of all positive integers. Many researchers have used Mann's iteration for obtaining weak convergence theorem (see for example [9–11]).

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In 2003, Nakajo and Takahashi [12] introduced a modification of the Mann iteration and called it CQ method. They generated the sequence $\{x_n\}$ by

$$\begin{cases} x_0 = x \in C, \\ y_n = \alpha_n x_n + (1-\alpha_n)Tx_n, \\ C_n = \{z \in C : \|y_n - z\| \le \|x_n - z\|\}, \\ Q_n = \{z \in C : \langle x_n - z, x_0 - x_n\rangle \le 0\}, \\ x_{n+1} = P_{C_n \cap Q_n} x_0, \quad n \ge 1, \end{cases} \tag{1.1}$$

for $n \in \mathbb{N} \cup \{0\}$, where $\{\alpha_n\} \subset [0, a]$ for some $a \in [0, 1)$. They showed that $\{x_n\}$ converges strongly to $P_{F(T)}x_0$.

In 2015, Khan et al. [13] used the following definition defined by Berinde [14] to compare the convergence rate:

Let $\{a_n\}$ and $\{b_n\}$ be two sequences of real numbers with limits a and b, respectively. Assume that there exists $\lim_{n\to\infty} \frac{|a_n - a|}{|b_n - b|} = \ell$. If $\ell = 0$, then we say that $\{a_n\}$ converges faster to a than $\{b_n\}$ to b.

Let C be a nonempty subset of a real Banach space X. Let $\triangle$ denote the diagonal of the cartesian product $C \times C$. Consider a directed graph G such that the set $V(G)$ of its vertices coincides with C, and the set $E(G)$ of its edges with $\triangle \subseteq E(G)$. We assume G has no parallel edge. So we can identify the graph G with the pair $(V(G), E(G))$. A mapping $T : C \to C$ is said to be

1. *G-contraction* if T satisfies the conditions:

(i) T preserves edges of G, *i.e.*,

$$(x, y) \in E(G) \Rightarrow (Tx, Ty) \in E(G), \ \forall (x, y) \in E(G);$$

(ii) T decreases weights of edges of G in the following way: there exists $\alpha \in (0, 1)$ such that

$$(x, y) \in E(G) \Rightarrow \|Tx - Ty\| \le \alpha\|x - y\|, \ \forall (x, y) \in E(G);$$

2. *G-nonexpansive* if T satisfies the conditions:

(i) T preserves edges of G, *i.e.*,

$$(x, y) \in E(G) \Rightarrow (Tx, Ty) \in E(G), \ \forall (x, y) \in E(G);$$

(ii) T non-increases weights of edges of G in the following way:

$$(x, y) \in E(G) \Rightarrow \|Tx - Ty\| \le \|x - y\|, \ \forall (x, y) \in E(G).$$

In 2008, Jachymski [15] proved some generalizations of the Banach's contraction principle in complete metric spaces endowed with a graph. To be more precise, Jachymski proved the following result.

Theorem 1.1 (*[15]*). *Let (X, d) be a complete metric space, and a triple (X, d, G) has the following property:*

for any sequence $\{x_n\}$ if $x_n \to x$ and $(x_n, x_{n+1}) \in E(G)$ for $n \in \mathbb{N}$ and there is the subsequence $\{x_{n_k}\}$ of $\{x_n\}$ with $(x_{n_k}, x) \in E(G)$ for $n \in \mathbb{N}$.

Let $T : X \to X$ be a G-contraction, and $X_T = \{x \in X : (x, Tx) \in E(G)\}$. Then $F(T) \neq \emptyset$ if and only if $X_T \neq \emptyset$.

When $\{x_n\}$ is a sequence in X, we denote the strong convergence of $\{x_n\}$ to $x \in X$ by $x_n \to x$ and the weak convergence by $x_n \rightharpoonup x$.

In 2015, Tiammee et al. [16] and Alfuraidan [17] employed the above theorem to establish the existence and the convergence results for G-nonexpansive mappings with graphs.

Motivated by Nakajo and Takahashi [12] and Tiammee et al. [16], we introduce the modified CQ method for proving a strong convergence theorem for G-nonexpansive mappings in a Hilbert space endowed with a directed graph. Moreover, we provide some numerical examples to support our main theorem.

2. Preliminaries and lemmas

Let C be a nonempty, closed and convex subset of a Hilbert space H. The nearest point projection of H onto C is denoted by P_C, that is, $\|x - P_C x\| \le \|x - y\|$ for all $x \in H$ and $y \in C$. Such P_C is called the *metric projection* of H onto C. We know that the metric projection P_C is firmly nonexpansive, i.e.,

$$\|P_C x - P_C y\|^2 \le \langle P_C x - P_C y, x - y\rangle$$

for all $x, y \in H$. Furthermore, $\langle x - P_C x, y - P_C x\rangle \le 0$ holds for all $x \in H$ and $y \in C$; see [18].

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