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On 3-total edge product cordial labeling of honeycomb

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Abstract

For a graph $G = (V(G), E(G))$, an edge labeling $\varphi : E(G) \to \{0, 1, \ldots, k-1\}$ where k is an integer, $2 \le k \le |E(G)|$, induces a vertex labeling $\varphi^* : V(G) \to \{0, 1, \ldots, k-1\}$ defined by $\varphi^*(v) = \varphi(e_1) \cdot \varphi(e_2) \cdot \ldots \cdot \varphi(e_n) \pmod k$, where $e_1, e_2, \ldots, e_n$ are the edges incident to the vertex v. The function φ is called a k-total edge product cordial labeling of G if $|(e_\varphi(i) + v_{\varphi^*}(i)) - (e_\varphi(j) + v_{\varphi^*}(j))| \le 1$ for every i, j, $0 \le i < j \le k-1$, where $e_\varphi(i)$ and $v_{\varphi^*}(i)$ are the number of edges e and vertices v with $\varphi(e) = i$ and $\varphi^*(v) = i$, respectively.

In this paper, we investigate the existence of 3-total edge product cordial labeling of hexagonal grid.

Keywords: Cordial labeling; k-total edge product cordial labeling; Hexagonal grid

1. Introduction

In this paper all graphs are finite, simple and undirected. Let $V(G)$ and $E(G)$ be the vertex set and the edge set of a graph G. We follow the basic notation and terminology of graph theory as in [1]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (or edges), then the labeling is called a *vertex labeling* (or an *edge labeling*). If the domain is $V(G) \cup E(G)$ then we call the labeling a *total labeling*.

A vertex labeling $\phi : V(G) \to \{0, 1\}$ induces an edge labeling $\phi^* : E(G) \to \{0, 1\}$ defined by $\phi^*(uv) = |\phi(u) - \phi(v)|$. For a vertex labeling ϕ and $i \in \{0, 1\}$, a vertex v is an i-vertex if $\phi(v) = i$ and an edge is an i-edge if $\phi^*(e) = i$. Denote the numbers of 0-vertices, 1-vertices, 0-edges, and 1-edges of G under ϕ and ϕ^* by $v_\phi(0)$, $v_\phi(1)$, $e_{\phi^*}(0)$, and $e_{\phi^*}(1)$, respectively. A vertex labeling ϕ is called *cordial* if $|v_\phi(0) - v_\phi(1)| \le 1$ and $|e_{\phi^*}(0) - e_{\phi^*}(1)| \le 1$.

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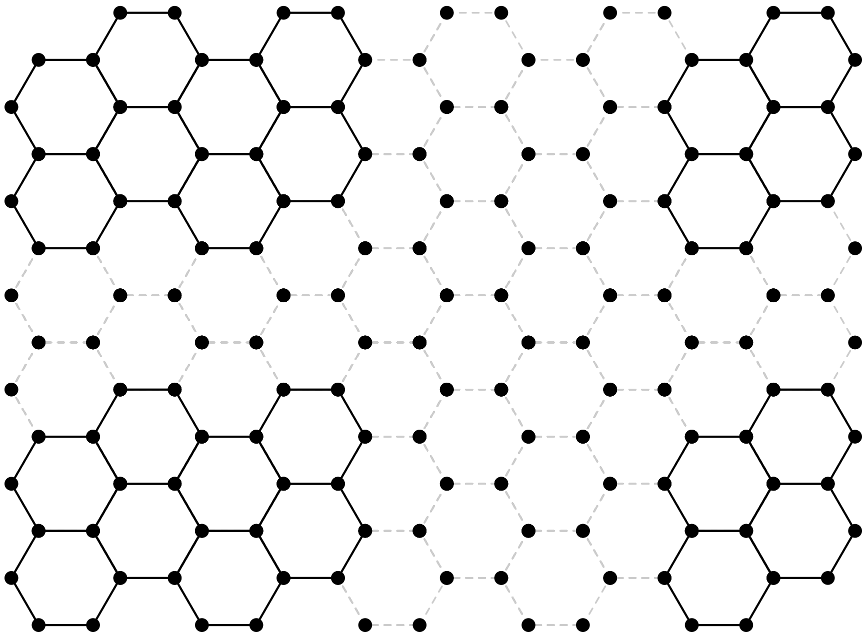

Fig. 1. The honeycomb H_n^m.

The notion of cordial labeling was first introduced by Cahit [2] as a weaker version of graceful labeling. See [3–5] for related results and [6–8] for generalizations. Cordial labelings of various families of graphs were studied in [9–11].

A binary vertex labeling $\phi : V(G) \to \{0, 1\}$ with induced edge labeling $\phi^* : E(G) \to \{0, 1\}$ defined by $\phi^*(uv) = \phi(u)\phi(v)$ is called a *product cordial labeling* if $|v_\phi(0) - v_\phi(1)| \le 1$ and $|e_{\phi^*}(0) - e_{\phi^*}(1)| \le 1$. The concept of product cordial labeling was introduced by Sundaram et al. [12]. Some labelings with variations in cordial theme, namely an edge product cordial labeling and a total edge product cordial labeling have been introduced by Vaidya and Barasara in [13,14].

Let k be an integer, $2 \le k \le |E(G)|$. An edge labeling $\varphi : E(G) \to \{0, 1, \dots, k-1\}$ with induced vertex labeling $\varphi^* : V(G) \to \{0, 1, \dots, k-1\}$ defined by $\varphi^*(v) = \varphi(e_1) \cdot \varphi(e_2) \cdot \cdots \cdot \varphi(e_n) \pmod k$, where $e_1, e_2, \dots, e_n$ are the edges incident to the vertex v, is called a *k-total edge product cordial labeling* of G if $|(e_\varphi(i) + v_{\varphi^*}(i)) - (e_\varphi(j) + v_{\varphi^*}(j))| \le 1$ for every $i, j, 0 \le i < j \le k-1$.

The concept of k-total edge product cordial labeling was introduced by Azaizeh et al. in [15]. A graph G with a k-total edge product cordial labeling is called k-total edge product cordial graph. In the next section, we investigate the existence of 3-total edge product cordial labeling for hexagonal grid.

2. Results

For $n \ge 1, m \ge 1$ we denote by H_n^m the hexagonal grid (honeycomb) as the planar graph with m rows and n columns of hexagons. Thus the hexagonal grid contains $2mn + 2(m + n)$ vertices, $3mn + 2(m + n) - 1$ edges, mn 6-sided faces and one external infinite face. Fig. 1 illustrates the honeycomb H_n^m for n even.

Next theorem shows that H_1^m and H_n^1 admit 3-total edge product cordial labeling for $m, n \ge 2$. Let us note that by *partial edge* we will mean the edge with only one end vertex.

Theorem 1. *The graphs H_1^m and H_n^1 are 3-total edge product cordial for $m \ge 2$, $n \ge 2$.*

Proof. Let $m = 1$.

Let us consider that H_1^1 admits 3-total edge product cordial labeling φ and let φ^* be the induced vertex labeling. Then, as H_1^1 is of order 6 and size 6, then in the set of all edge labels and induced vertex labels every number of 0, 1 and 2 must be used exactly 4 times. Vertex label of a vertex v can be 0 only if at least one edge incident with v is labeled with 0. However, the label of the other end vertex of this edge will be labeled with 0 in this case. This leads to contradiction that number of elements labeled with 0 must be 4.