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Edge odd graceful labeling of some path and cycle related graphs

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Abstract

Solairaju and Chithra introduced a new type of labeling of a graph G with p vertices and q edges called an edge odd graceful labeling if there is a bijection f from the edges of the graph to the set $\{1, 3, \ldots, 2q-1\}$ such that, when each vertex is assigned the sum of all edges incident to it $\bmod 2k$, where $k = \max(p, q)$, the resulting vertex labels are distinct. In this paper we proved necessary and sufficient conditions for some path and cycle related graphs to be edge odd graceful such as: Friendship graphs, Wheel graph, Helm graph, Web graph, Double wheel graph, Gear graph, Fan graph, Double fan graph and Polar grid graph.

Keywords: Edge odd graceful labeling; Friendship graph; Wheel graph; Double fan graph; Polar grid graph

1. Introduction

The field of Graph Theory plays an important role in various areas of pure and applied sciences. Graph Labeling of a graph G is an assignment of integers either to the vertices or edges or both subject to certain conditions. Graph labeling is a very powerful tool that eventually makes things in different fields very ease to be handled in mathematical way. Nowadays graph labeling has much attention from different brilliant researches in graph theory which has rigorous applications in many disciplines, e.g., communication networks, coding theory, optimal circuits layouts, astronomy, radar and graph decomposition problems. See [1–3].

We begin with simple, connected, finite, undirected graph $G = (V(G), E(G))$ with $p = |V(G)|$ and $q = |E(G)|$.

In 1967, Rosa [4] introduced a labeling of G called β-valuation, later on Solomon W. Golomb [5] called as "graceful labeling" which is an injection f from the set of vertices $V(G)$ to the set $\{0, 1, 2, \ldots, q\}$ such that when each edge $e = uv$ is assigned the label $|f(u) - f(v)|$, the resulting edge labels are distinct. A graph which admits a graceful labeling is called a graceful graph.

In 1991, Gnanajothi [6] introduced a labeling of G called odd graceful labeling which is an injection f from the set of vertices $V(G)$ to the set $\{0, 1, 2, \ldots, 2q-1\}$ such that when each edge $e = uv$ is assigned the label $|f(u) - f(v)|$,

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the resulting edge labels are $\{1, 3, \ldots, 2q - 1\}$. A graph which admits an odd graceful labeling is called an odd graceful graph.

In 1985, Lo [7] introduced a labeling of G called edge graceful labeling, which is a bijection f from the set of edges $E(G)$ to the set $\{1, 2, \ldots, q\}$ such that the induced map f^* from the set of vertices $V(G)$ to $\{0, 1, 2, \ldots, p-1\}$ given by $f^*(u) = \sum_{uv \in E(G)} f(uv) (\text{mod } p)$ is a bijection. A graph which admits edge graceful labeling is called an edge graceful graph.

In 2009, Solairaju and Chithra [8] introduced a labeling of G called edge odd graceful labeling, which is a bijection f from the set of edges $E(G)$ to the set $\{1, 3, \ldots, 2q-1\}$ such that the induced map f^* from the set of vertices $V(G)$ to $\{0, 1, 2, \ldots, 2q-1\}$ given by $f^*(u) = \sum_{uv \in E(G)} f(uv) (\text{mod } 2q)$ is a bijection. A graph which admits edge odd graceful labeling is called an edge odd graceful graph.

2. Edge odd graceful labeling of friendship graph $Fr_n^{(3)}$

The friendship graph $Fr_n^{(3)}$, is a planar undirected graph with $2n+1$ vertices and $3n$ edges constructed by joining n copies of the cycle graph C_3 with a common vertex.

Theorem 1. *The friendship graph $F_{r_n}^{(3)}$, is an edge odd graceful graph.*

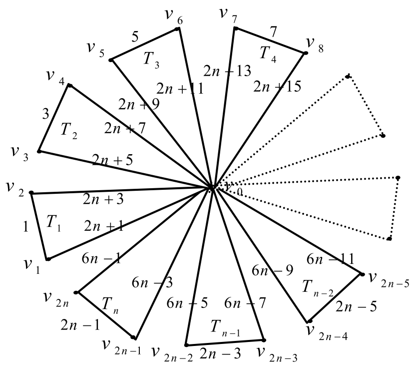

Fig. 1. Friendship graph $Fr_n^{(3)}$, $n \equiv 0\,(\text{mod } 3)$.

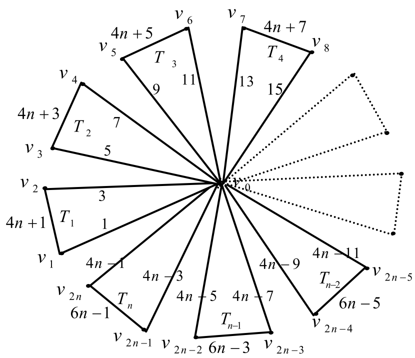

Fig. 2. Friendship graph $Fr_n^{(3)}$, $n \equiv 1\,(\text{mod } 3)$.

Proof. Using standard notation $p = |V(G)| = 2n+1$, $q = |E(G)| = 3n$ and $k = \max(p, q) = 3n$. There are three cases:

Case (1): $n \equiv 0\,(\text{mod } 3)$. Let the friendship graph $F_{r_n}^{(3)}$, be as in Fig. 1 and the triangles in it are $T_1, T_2, \ldots, T_n$. Name the center by v_0 and name the other two vertices of T_i by v_{2i-1}, v_{2i}, $i = 1, 2, \ldots, n$. First label the outer edges, beginning from the base of triangle T_1 to the triangle T_n by $1, 3, \ldots, 2n-1$, then label the inner edges incident to the center (hub) by $2n+1, 2n+3, \ldots, 6n-1$. Then we obtain the labels of vertices v_i, $i = 1, 2, \ldots, n$ as follows : $2n+2, 2n+4;\ 2n+8, 2n+10; \ldots;\ 2n-4, 2n-2\,(\text{mod } 6n)$.

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