ARTICLE IN PRESS

Available online at www.sciencedirect.com

ScienceDirect

AKCE International Journal of Graphs and Combinatorics ▮ (▮▮▮▮) ▮▮▮–▮▮▮

AKCE International Journal of Graphs and Combinatorics

www.elsevier.com/locate/akcej

Local coloring of self complementary graphs

P. Deepa[a], P. Srinivasan[b], M. Sundarakannan[c,*]

[a] *Department of Mathematics, Prathyusha Institute of Technology and Management, Aranvoyal kuppam, Ponamalee, Thiruvallur - 602 025, Tamil Nadu, India*

[b] *Department of Mathematics, The American College, Madurai - 625 002, Tamil Nadu, India*

[c] *Department of Mathematics, SSN College of Engineering, Kalavakkam - 603 110, Chennai, Tamil Nadu, India*

Received 10 June 2015; accepted 12 May 2016

Available online xxxx

Abstract

Let $G = (V, E)$ be a graph. A *local coloring* of a graph G of order at least 2 is a function $c : V(G) \longrightarrow \mathbb{N}$ having the property that for each set $S \subseteq V(G)$ with $2 \leq |S| \leq 3$, there exist vertices $u, v \in S$ such that $|c(u) - c(v)| \geq m_s$, where m_s is the size of the induced subgraph $\langle S \rangle$. The maximum color assigned by a local coloring c to a vertex of G is called the value of c and is denoted by $\chi_\ell(c)$. The *local chromatic number* of G is $\chi_\ell(G) = \min\{\chi_\ell(c)\}$, where the minimum is taken over all local colorings c of G. In this paper we study the local coloring for some self complementary graphs. Also we present a sc-graph with local chromatic number k for any given integer $k \geq 6$.

Keywords: Coloring; Local coloring; Local chromatic number; Self complementary graph

1. Introduction

By a graph $G = (V, E)$ we mean a finite and undirected graph with neither loops nor multiple edges. The order and size of G are denoted by n and m respectively. For graph theoretic terminology we refer to Chartrand and Lesniak [1].

Graph coloring is one of the major areas in graph theory that have been well studied. Several variations of coloring have been introduced and studied by many researchers. Excellent books of [2,3] give an extensive survey of various graph colorings and many open problems.

Let $v \in V$. The *open neighborhood* of v denoted by $N(v)$ and the *closed neighborhood* of v denoted by $N[v]$ are defined by $N(v) = \{u \in V : uv \in E\}$ and $N[v] = N(v) \cup \{v\}$. The disjoint *union* of two graphs G and H is $G \cup H$ with vertex set $V(G) \cup V(H)$ and edge set $E(G) \cup E(H)$. The *join* $G + H$ consists of $G \cup H$ and all edges joining a vertex of G and a vertex of H. A graph G is said to be *self complementary* (sc-graph) if G is isomorphic to its

Peer review under responsibility of Kalasalingam University.

* Corresponding author.

E-mail addresses: deepap79@gmail.com (P. Deepa), srinivasanp.ac@gmail.com (P. Srinivasan), m.sundarakannan@gmail.com (M. Sundarakannan).

http://dx.doi.org/10.1016/j.akcej.2016.11.005

complement $\overline{G}$. A *proper coloring* of a graph G is an assignment of colors to the vertices of G in such a way that no two adjacent vertices receive the same color. The *chromatic number* $\chi(G)$ is the minimum number of colors required for a proper coloring of G. The standard definition of coloring can also be rephrased as below. For a graph G and a nonempty subset $S \subseteq V(G)$, let m_s denote the size of the induced subgraph $\langle S \rangle$. A standard coloring of a graph G can be considered as a function $c : V(G) \longrightarrow \mathbb{N}$ with the property that for every 2-element set $S = \{u, v\}$ of vertices of G, $|c(u) - c(v)| \geq m_s$.

Motivated by these observations, Chartrand et al. [4,5] introduced the following concept of local coloring and local chromatic number. Omoomi et al. [6–8] recently proved many substantial results based on the local coloring of graphs, Kneser graphs and locally rainbow graphs. Let $\mathbb{N}[t] = \{1, 2, \ldots, t\}$. For $k \geq 2$, a *k-local coloring* of a graph G of order at least 2 is a function $c : V(G) \longrightarrow \mathbb{N}[t]$ having the property that for each set $S \subseteq V(G)$ with $2 \leq |S| \leq k$, there exist vertices $u, v \in S$ such that $|c(u) - c(v)| \geq m_s$, where m_s is the size of the induced subgraph $\langle S \rangle$. The maximum color assigned by a k-local coloring c to a vertex of G is called the value of c and is denoted by $l_{c_k}(c)$. The *k-local chromatic number* of G is $l_{c_k}(G) = \min\{l_{c_k}(c)\}$, where the minimum is taken over all k-local colorings c of G. Even though we have defined k-local coloring and k-local chromatic number, hereafter we will consider only a 3-local coloring c of a graph G which is referred to as a *local coloring* of G and $l_{c_3}(c)$ is denoted by $\chi_\ell(c)$ and $l_{c_3}(G)$ is denoted by $\chi_\ell(G)$, which is referred to as *local chromatic number* of G. If $\chi_\ell(c) = \chi_\ell(G)$, then c is called a minimum local coloring of G.

Chartrand et al. [5] posed the following problem: Does there exist a graph G_k such that $\chi(G_k) = \chi_\ell(G_k) = k \geq 5$? This unsolved problem shows the hardness of the local coloring parameter and so far there is no proof technique to determine for $\chi_\ell(G_k) = k \geq 4$. Self-complementary graphs have been extensively investigated and effectively used in the study of Ramsey numbers. It is well-known and easily proved that there exist regular self-complementary graphs of order n if and only if $n \equiv 1 \pmod 4$. So it is natural to ask whether a similar result for local chromatic number for self-complementary graph exists.

We need the following sc-graph construction. For sc-graph construction we refer to [9]. Consider two copies of any graph G, say G_1, G_4 and two copies of $\overline{G}$ say G_2, G_3. Now join every vertex of G_1 to every vertex of G_2, every vertex of G_2 to every vertex of G_3 and every vertex of G_3 to every vertex of G_4. The resulting graph is a sc-graph and it is called *P_4-construction* obtained from the graph G and is denoted by $P(G)$. That is,

$$P(G) = G_1 \sim G_2 \sim G_3 \sim G_4$$
$$\overline{P(G)} = G_2 \sim G_4 \sim G_1 \sim G_3.$$

Also we need the following results.

Proposition 1.1 ([4,5]).

1. *For the complete graph K_n, $\chi_\ell(K_n) = \left\lfloor \frac{3n-1}{2} \right\rfloor$.*
2. *Let n be even and let $\chi_\ell(K_n) = k$. Then K_n has unique k-local coloring.*

Lemma 1.2 ([7]). *For every two subgraphs G and H, we have $\chi_\ell(G + H) \leq \chi_\ell(G) + \chi_\ell(H) + 1$.*

Several results on local chromatic number have been proved by many researchers in [6,10,11,9,12] focusing on particular classes of graphs like Kneser graph, Mycielskian of graphs, regular graphs, Peterson graphs and permutation graphs. Classes of self-complementary graphs rarely appear in investigations concerning coloring parameters. In this paper, we consider family of self-complementary graphs. To be specific we present the local chromatic number of sc-graph $P(G)$ for a given graph G. We then study the local chromatic number of sc-graph G with $\chi_\ell(G) = k$ for a given integer $k \geq 6$ and also determine the local chromatic number of sc-graphs of order up to 8.

2. Local coloring of self complementary graphs

In this section we present bounds for the local chromatic number of a sc-graph $P(G)$ and prove some related results. Also we present a construction of a sc-graph with local chromatic number for any given integer $k \geq 6$. We consider the following notions throughout this section. Let $V(P(G)) = V_1 \cup V_2 \cup V_3 \cup V_4$, where $V_i = V(G_i) = \{v_{ij} \mid j = 1, 2, \ldots, n\}$ for $i = 1, 2, 3, 4$.

Please cite this article in press as: P. Deepa, et al., Local coloring of self complementary graphs, AKCE International Journal of Graphs and Combinatorics (2016), http://dx.doi.org/10.1016/j.akcej.2016.11.005