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On the edge irregularity strength of corona product of cycle with isolated vertices

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Abstract

In this paper, we investigate the new graph characteristic, the edge irregularity strength, denoted as *es*, as a modification of the well known irregularity strength, total edge irregularity strength and total vertex irregularity strength. As a result, we obtain the exact value of an edge irregularity strength of corona product of cycle with isolated vertices.

Keywords: Irregularity assignment; Irregularity strength; Edge irregularity strength; Unicyclic graphs

1. Introduction

Let G be a connected, simple and undirected graph with vertex set $V(G)$ and edge set $E(G)$. By a *labeling* we mean any mapping that maps a set of graph elements to a set of numbers (usually positive integers), called *labels*. If the domain is the vertex-set or the edge-set, the labelings are called respectively *vertex labelings* or *edge labelings*. If the domain is $V(G) \cup E(G)$, then we call the labeling *total labeling*. Thus, for an edge k-labeling $\delta : E(G) \to \{1, 2, \dots, k\}$ the associated weight of a vertex $x \in V(G)$ is

$$w_\delta(x) = \sum \delta(xy),$$

where the sum is over all vertices y adjacent to x.

Chartrand et al. [1] introduced edge k-labeling δ of a graph G such that $w_\delta(x) = \sum \delta(xy)$ for all vertices $x, y \in V(G)$ with $x \neq y$. Such labelings were called *irregular assignments* and the *irregularity strength* $s(G)$ of a graph G is known as the minimum k for which G has an irregular assignment using labels at most k. This parameter has attracted much attention [2–8].

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Motivated by these papers, Baca et al. [9] defined a *vertex irregular total k-labeling* of a graph G to be a total labeling of G, $\psi : V(G) \cup E(G) \to \{1, 2, \ldots, k\}$, such that the *total vertex-weights*

$$wt(x) = \psi(x) + \sum_{xy \in E(G)} \psi(xy)$$

are different for all vertices, that is, $wt(x) \neq wt(y)$ for all different vertices $x, y \in V(G)$. The *total vertex irregularity strength* of G, $tvs(G)$, is the minimum k for which G has a vertex irregular total k-labeling. They also defined the total labeling $\psi : V(G) \cup E(G) \to \{1, 2, \ldots, k\}$ to be an edge irregular total k-labeling of the graph G if for every two different edges xy and $x'y'$ of G one has $wt(xy) = \psi(x) + \psi(xy) + \psi(y) \neq wt(x'y') = \psi(x') + \psi(x'y') + \psi(y')$. The *total edge irregularity strength*, $tes(G)$, is defined as the minimum k for which G has an edge irregular total k-labeling. Some results on the total vertex irregularity strength and the total edge irregularity strength can be found in [10–16,8,17–19].

The most complete recent survey of graph labelings is [20].

A vertex k-labeling $\phi : V(G) \to \{1, 2, \ldots, k\}$ is called an *edge irregular k-labeling* of the graph G if for every two different edges e and f, there is $w_\phi(e) \neq w_\phi(f)$, where the weight of an edge $e = xy \in E(G)$ is $w_\phi(xy) = \phi(x) + \phi(y)$. The minimum k for which the graph G has an edge irregular k-labeling is called the *edge irregularity strength* of G, denoted by $es(G)$ (see [21]).

In [21], the authors estimated the bounds of the edge irregularity strength es and then determined its exact values for several families of graphs namely, paths, stars, double stars and Cartesian product of two paths. Mushayt [22] determined the edge irregularity strength of cartesian product of star, cycle with path P_2 and strong product of path P_n with P_2. Tarawneh et al. [23] investigated the edge irregularity strength of corona product of graph with paths. Recently, Ahmad [24] determined the exact value of the edge irregularity strength of corona graph $C_n \odot K_1$ (or sun graph S_n).

The following theorem established lower bound for the edge irregularity strength of a graph G.

Theorem 1 *([21]). Let $G = (V, E)$ be a simple graph with maximum degree $\Delta = \Delta(G)$. Then*

$$es(G) \geq \max\left\{\left\lceil \frac{|E(G)| + 1}{2} \right\rceil, \Delta(G)\right\}.$$

In this paper, we determine the exact value of edge irregularity strength of corona graphs $C_n \odot mK_1$, $m \geq 2$.

2. Two lemmas

The corona product of two graphs G and H, denoted by, $G \odot H$, is a graph obtained by taking one copy of G (which has n vertices) and n copies $H_1, H_2, \ldots, H_n$ of H, and then joining the ith vertex of G to every vertex in H_i.

The corona product $C_n \odot mK_1$ is a graph with the vertex set $V(C_n \odot mK_1) = \{x_i, y_i^j : 1 \leq i \leq n, 1 \leq j \leq m\}$ and edge set $E(C_n \odot mK_1) = \{x_i x_{i+1} : 1 \leq i \leq n-1\} \cup \{x_i y_i^j : 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{x_n x_1\}$ is the edge set of $C_n \odot mK_1$. The corona product $C_n \odot K_1$ is also know as sun graph S_n. Ahmad [24] determined the exact value of the edge irregularity strength of $C_n \odot K_1 = S_n$.

The following two lemmas determine the exact value of the edge irregularity strength for two particular cases.

Lemma 1. *Let $C_n \odot 2K_1$, $n \geq 3$, be a corona graph. Then, $es(C_n \odot 2K_1) = \lceil \frac{3n+1}{2} \rceil$.*

Proof. The graph $C_n \odot 2K_1$ has $3n$ vertices and $3n$ edges. The maximum degree of $\Delta(C_n \odot 2K_1)$ is 4. Therefore, by Theorem 1, we have that $es(C_n \odot 2k_1) \geq \max\left\{\left\lceil \frac{3n+1}{2} \right\rceil, 4\right\} = \lceil \frac{3n+1}{2} \rceil$. To prove the equality, it suffices to prove the existence of an optimal edge irregular $\lceil \frac{3n+1}{2} \rceil$−labeling. Assume $k = \left\lceil \frac{3n+1}{2} \right\rceil$. Let $\phi_1 : V(C_n \odot 2K_1) \to \{1, 2, \ldots, \lceil \frac{3n+1}{2} \rceil\}$ be the vertex labeling such that:

For $1 \leq i \leq \lfloor \frac{n}{2} \rfloor + 1$, $\phi_1(x_i) = 2\lceil \frac{i-1}{2} \rceil + \lceil \frac{i}{2} \rceil$ and for $\lfloor \frac{n}{2} \rfloor + 2 \leq i \leq n$, $\phi_1(x_i) = k - 2\lceil \frac{n-i}{2} \rceil - \lfloor \frac{n-i}{2} \rfloor$. $\phi_1(y_i^j) = 2\lfloor \frac{i-1}{2} \rfloor + \lfloor \frac{i}{2} \rfloor + j$ for $1 \leq i \leq \lfloor \frac{n}{2} \rfloor$, $1 \leq j \leq 2$. For $i = \lfloor \frac{n}{2} \rfloor + 1$, $n \equiv 1 \pmod 4$, $1 \leq j \leq 2$, $\phi_1(y_i^j) = \frac{3n-11}{4} + 3j$.