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Note

Approximately locating an invisible agent in a graph with relative distance queries

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ABSTRACT

In a pursuit evasion game on a finite, simple, undirected, and connected graph G, a first player visits vertices $m_1, m_2, \dots$ of G, where m_{i+1} is in the closed neighborhood of m_i for every i, and a second player probes arbitrary vertices $c_1, c_2, \dots$ of G, and learns whether or not the distance between c_{i+1} and m_{i+1} is at most the distance between c_i and m_i. Up to what distance d can the second player determine the position of the first? For trees of bounded maximum degree and grids, we show that d is bounded by a constant. We conjecture that $d = O(\log n)$ for every graph G of order n, and show that $d = 0$ if m_{i+1} may differ from m_i only if i is a multiple of some sufficiently large integer.

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1. Introduction

We study a variant of pursuit and evasion games formalized and studied by Britnell and Wildon [2], Komarov and Winkler [6], Haslegrave [4], Seager [8], and Rautenbach and Schneider [7]. In these games, further studied in [1,3,5,9], one player tries to catch or locate a second player moving along the edges of a graph, using information concerning the current position of the second player. Our game is also played by two players on a finite, simple, undirected, and connected graph G known to both of them, and proceeds in discrete time steps numbered by positive integers. One player, called the *mouse*, moves along the edges of G. At time i, the mouse occupies some vertex m_i of G, and, if i is at least 2, then m_i is either m_{i-1} or a neighbor of m_{i-1}, that is, m_i belongs to the closed neighborhood $N_G[m_{i-1}]$ of m_{i-1} in G, and the mouse can be considered to move with unit speed. The second player, called the *cat*, probes vertices of G one by one in the same discrete time steps. At time i, the cat probes some vertex c_i of G, where c_i can be chosen without any restriction within the vertex set $V(G)$ of G.

The essential difference of our game, as compared to those mentioned above, consists in the information provided to the cat. If i is at least 2, then, after m_i and c_i have been decided by the two players, the cat learns whether

- $d_i \leq d_{i-1}$ or
- $d_i > d_{i-1}$,

where d_i denotes the distance $\text{dist}_G(c_i, m_i)$ in G between c_i and m_i. Note that unlike in some of the settings considered in the references, the mouse may choose not to move, and the cat does not automatically locate the mouse if $m_i = c_i$. The goal of the cat is to locate the mouse as precisely as possible, while the goal of the mouse is to hinder being well located. To make this more precise, we introduce some further terminology.

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A *game* g on G is a pair of sequences $((m_i)_{i\in\mathbb{N}}, (c_i)_{i\in\mathbb{N}})$ of possible moves m_i for the mouse, and c_i for the cat. For such a game g, and an integer i at least 2, let

$$b_i = \begin{cases} 1 & \text{, if } d_i \leq d_{i-1}, \text{ and} \\ 0 & \text{, if } d_i > d_{i-1}, \end{cases}$$

that is, the information available to the cat for its choice of c_{i+1} consists of G and the $i-1$ *bits* $b_2, \ldots, b_i$. Note that the cat chooses c_1 and c_2 without any information about the whereabouts of the mouse. Based on the available information, the cat knows that m_i belongs to the set M_i, where M_i is the set of all vertices u of G such that there are vertices $\tilde{m}_1, \ldots, \tilde{m}_i$ of G with

- $\tilde{m}_i = u$,
- $\tilde{m}_j \in N_G[\tilde{m}_{j-1}]$ for every $j \in [i] \setminus \{1\}$, and
- $\text{dist}_G(c_j, \tilde{m}_j) \leq \text{dist}_G(c_{j-1}, \tilde{m}_{j-1})$ if and only if $b_j = 1$ for every $j \in [i] \setminus \{1\}$,

where $[i]$ denotes the set of positive integers at most i.

If the *radius* $\text{rad}_G(M)$ of a set M of vertices of G is defined as

$$\min\Big\{\max\Big\{\text{dist}_G(u, m) : m \in M\Big\} : u \in V(G)\Big\},$$

then the cat wants to minimize the radius of M_i. Note that the vertex u in this definition might not belong to M.

We say that the cat follows a *strategy* $(c_1, c_2; f)$ in the game $((m_i)_{i\in\mathbb{N}}, (c_i)_{i\in\mathbb{N}})$ on G if c_1 and c_2 are vertices of G, and

$$f : \bigcup_{i\in\mathbb{N}} \{0, 1\}^i \to V(G)$$

is a function such that c_1 and c_2 are the two first vertices probed by the cat, and $c_{i+1} = f(b_2, \ldots, b_i)$ for every integer i at least 2. Furthermore, we say that *the cat can localize the mouse up to distance d within time t on G* if there is some strategy σ such that for every game $((m_i)_{i\in\mathbb{N}}, (c_i)_{i\in\mathbb{N}})$ on G in which the cat follows the strategy σ, there is some positive integer i at most t with $\text{rad}_G(M_i) \leq d$.

While the cat only knows G and the b_i, and therefore also the set M_i, we may assume that the mouse knows G and also any strategy followed by the cat. Note that we consider a game to be infinite, and that we did not specify any winning conditions. A reasonable way to do so is to fix a distance threshold d, and to declare the cat to be the winner on the pair (G, d) if it can locate the mouse up to distance d within finite time.

A natural question concerning our game is how precisely the cat can localize the mouse on a given graph. Our first result provides an answer for trees of bounded maximum degree.

Theorem 1.1. *The cat can localize the mouse up to distance $4\Delta - 6$ within time $O(h\Delta)$ on every tree T of maximum degree Δ at least 2 and radius h.*

Another natural type of graphs to consider are grids, that is, the Cartesian product $P_n \square P_m$ of paths. For these we show the following.

Theorem 1.2. *The cat can localize the mouse up to distance 8 within time $O(\log n)$ on the grid $P_n \square P_n$.*

Both our results concern graphs of bounded maximum degree, and we pose the following general conjecture.

Conjecture 1.3. *The cat can localize the mouse up to distance $O(\log n)$ on a connected graph G of order n.*

The reason for the $O(\log n)$ term in this conjecture is that this many bits suffice to identify each vertex, while the mouse may move this many units of distance in the time needed to acquire this many bits.

Our final result establishes a weak version of Conjecture 1.3. In order to facilitate the task for the cat, we slow down the mouse as follows. For some positive integer k, a game $((m_i)_{i\in\mathbb{N}}, (c_i)_{i\in\mathbb{N}})$ on a graph G is *k-slow* if $m_i = m_{i-1}$ for every integer i at least 2 such that $(i-1) \not\equiv 0 \mod k$, that is, the mouse can be considered to move with speed $1/k$. We say that *the cat can localize a k-slow mouse up to distance d on G* if there is some strategy σ such that for every k-slow game $((m_i)_{i\in\mathbb{N}}, (c_i)_{i\in\mathbb{N}})$ on G in which the cat follows the strategy σ, there is some positive integer i with $\text{rad}_G(M_i) \leq d$.

Theorem 1.4. *If Δ is an integer at least 2, then the cat can localize a 4Δ-slow mouse up to distance 0 on every connected graph G of maximum degree at most Δ.*

In Section 2 we prove our results, and in Section 3 we present some open problems.

2. Proofs

For all three of our results, we give simple proofs capturing essential observations. For Theorems 1.1 and 1.2, minor improvements are possible at the cost of tedious case analysis.

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