Available online at www.sciencedirect.com

ScienceDirect

Electronic Notes in DISCRETE MATHEMATICS

Electronic Notes in Discrete Mathematics 66 (2018) 103–110 www.elsevier.com/locate/endm

Variable Neighborhood Descent applied to Multi-way Number Partitioning Problem

Alexandre Frias Faria [a,1] Sérgio Ricardo de Souza [a,2]
Carlos Alexandre Silva [b,3]

[a] *CEFET-MG, Av. Amazonas 7675, 30510-000, Nova Gameleira, Belo Horizonte, MG, Brasil*

[b] *IFMG, Av. Serra da Piedade 299, 34515-640, Morada da Serra, Sabará, MG, Brasil*

Abstract

This paper presents an algorithm for the optimization version of the Multi-Way Number Partitioning Problem (MWNPP). This problem consists in distributing the elements of a given sequence into k disjoint subsets so that the sums of each subset elements fit in the shortest interval. The metaheuristic Variable Neighborhood Descent (VND), a deterministic variant of Variable Neighborhood Search (VNS), adapted for solving the MWNPP, has a good performance over instances less than six subsets. It is carried out a comparative study with two algorithms, Karmarkar-Karp Heuristic and Longest Processing Time, using randomly generated instances and objective functions values. The statistical tests show that results of the VND proposed are significantly better than literature constructive methods and its improvements.

Keywords: Combinatorial Optimization, Multi-way Number Partitioning Problem, Metaheuristic.

https://doi.org/10.1016/j.endm.2018.03.014

1 Introduction

In this article, a partition of a set X is a collection of pairwise disjoint non-empty subsets whose union forms X. A k-partition is a partition having exactly k non-empty subsets. The subsets belonging to the partition are called parts. The set $\mathbb{Z}_+$ indicates strictly positive integers numbers and the notation $I_m = \{y \in \mathbb{Z} \ : \ 1 \leq y \leq m\}$ denotes the set of all integers between 1 and m, inclusive.

This paper addresses the Multi-Way Number Partitioning Problem, in the sequel named MWNPP, first level generalization of the Two-Way Number Partitioning Problem (TWNPP). Let V be a numerical sequence. In the MWNPP, the objective is to find a k-partition of the indexes of V, so that the sums of the elements of each part are as close as possible to each other. This is equivalent to having the largest sum part as close to the smallest part as possible. An immediate conclusion is that, in relation to TWNPP, MWNPP expands the number of parts in which the elements of the V sequence must be distributed.

There is an extensive literature on TWNPP and its variations. It is a classical combinatorial optimization problem, formally listed in [6] as one of the basic NP-Complete problems. MWNPP arises explicitly in an article dealing with the analysis of a constructive heuristic called Differencing Method, better known as Karmarkar-Karp Heuristic (KKH), proposed by [5]. This heuristic seeks to divide the largest numbers of the sequence V into distinct parts, inserting, in the set of unallocated elements, the differences between the elements removed, as long as it is not empty. In [11], an approximation ratio given by $[4/3 - 1/(3(k-1))]$ is shown for KKH. This result imposes a limited error interval for the values obtained with KKH, being similar to the ratio found for the Longest Processing Time (LPT) algorithm, proposed in [3].

According to [1], MWNPP is a very difficult problem to be solved by general-use metaheuristics, like Genetic Algorithms, Simulated Annealing and others. In many cases, these methods lose in terms of computational time and in performance for the KKH and even for the LPT. The construction of exact algorithms is proposed in [7], where a backtrack procedure is performed in constructive heuristics, like LPT and KKH, called Complete Greedy Algorithm and Complete Karmarkar-Karp Algorithm, respectively. The first improvement in these works happens with the Recursive Number Partitioning Algorithm, proposed by [8]. The second improvement is the contribution of [13],

[1] Email: alexandrefrias1@hotmail.com
[2] Email: sergio@dppg.cefetmg.br
[3] Email: carlos.silva@ifmg.edu.br