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A hybrid VNS algorithm for solving the multi-level capacitated minimum spanning tree problem

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Abstract

This work addresses the multi-level capacitated minimum spanning tree (MLCMST) problem. It consists of finding a minimal cost spanning tree such that the flow to be transferred from a central node (root) to the other nodes is bounded by the edge capacities. In this paper, a hybrid algorithm, combining the Variable Neighborhood Search (VNS) metaheuristic and one mathematical programming formulation of the literature, is used for solving it. The formulation is used to give an initial solution to VNS. Five neighborhoods are used for exploring the solution space. Results show that the VNS is able to improve the initial solutions and to obtain small gap solutions for all instance sets.

Keywords: Multi-level capacitated minimum spanning tree, Variable Neighborhood Search, Combinatorial Optimization, Neighborhood structures

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1 Introduction

The multi-level capacitated minimum spanning tree (MLCMST) problem consists of finding a minimum cost spanning tree such that the flow to be transferred from a central node to the other nodes is bounded by the edge capacities. The MLCMST problem has been introduced in [4,3] and it is a natural extension of the capacitated minimum spanning tree (CMST) problem [2].

The possibility of having different capacities and costs for each edge mimics the concept of economies of scale, broadening thus the applicability of the MLCMST problem to communication networks [5] and logistic systems [1]. Due to its importance, some authors have addressed the MLCMST by different approaches. Gamvros et al. [4], Martins et al. [7], and Gamvros et al. [3] compared different formulations for the MLCMST problem. The one referred as the capacity-indexed model was considered to be the most efficient since it was the only one able to solve instances up to 30 nodes in the pre-established time limit of one hour [7]. Gamvros et al. [3] also devised a solution construction phase based upon savings and two local search procedures which rely on the cyclic and path exchange neighborhoods. The developed algorithms obtained average gaps from the lower bounds ranging from 6% to 9.9% in instances having up to 150 nodes.

Martins et al. [6] proposed a GRASP with a hybrid heuristic-subproblem optimization approach. By using heuristic rules both in the construction and the local search phases, smaller subproblems are defined and exactly solved by using a commercial package. This scheme allowed them to improve several best known upper bounds for 250 benchmark instances from the literature.

Uchoa et al. [9] proposed a Branch-and-Cut algorithm introducing two kinds of cuts: the exact separation of cuts corresponding to some master equality polyhedra found in the formulation and the separation of Fenchel cuts. Numerical results showed that the best known upper bounds were improved for almost all instances that could not be solved to optimality.

In this paper, a hybrid algorithm, combining the Variable Neighborhood Search (VNS) metaheuristic and one mathematical programming formulation of the literature, is used for solving MLCMST. The formulation is used to give an initial solution to the VNS. Five neighborhoods are used for exploring the solution space. Results show that the VNS is able to improve the initial solutions and to obtain small gaps solutions for all instance sets.

2 Definitions and formulation

Let $G(V, E)$ be the undirected graph, where V and E are the sets of nodes and edges, and L be the ordered index set of available types of connections. For