Available online at www.sciencedirect.com
ScienceDirect
Electronic Notes in DISCRETE MATHEMATICS
Electronic Notes in Discrete Mathematics 64 (2018) 5–14
www.elsevier.com/locate/endm

# A Branch-and-Bound Algorithm for the Minimum Cost Bipartite Perfect Matching Problem with Conflict Pair Constraints [1]

Temel Öncan [2]

*Endüstri Mühendisliği Bölümü, Galatasaray Üniversitesi, İstanbul, TÜRKİYE*

İ. Kuban Altınel [3]

*Endüstri Mühendisliği Bölümü, Boğaziçi Üniversitesi, İstanbul, TÜRKİYE*

**Abstract**

In this study we consider the Minimum Cost Bipartite Perfect Matching Problem with Conflict Pair Constraints (MCBPMPC) on bipartite graphs. Given a bipartite graph $G$ with a cost associated with each edge and a conflict set of edge pairs, the MCBPMPC consists of finding a perfect matching with the lowest total cost such that no more than one edge is selected from each pair in the conflict set. A specially tailored branch-and-bound (B&B) algorithm is introduced. Computational experiments are performed on randomly generated instances. According to the extensive experiments, the suggested B&B algorithm yields promising results compared to a state-of-the-art mixed integer linear programming solver.

*Keywords:* branch and bound, bipartite perfect matching problem, conflict graph, assignment problem

https://doi.org/10.1016/j.endm.2018.01.002

# 1 Introduction

A matching $M$ in a graph $G$ is defined as a set of edges such that no two edges of $M$ are incident on the same node. The matching problem and its several variations are well studied in the combinatorial optimization literature ([1]). Some variations of the matching problem such as the Maximum Cardinality Bipartite Matching Problem and the Minimum Cost Bipartite Perfect Matching Problem (MCBPMP) are polynomially solvable while other variations, such as the well-known Quadratic Assignment Problem (QAP) ([5]), are $\mathcal{NP}$-hard. For a thorough review on the bipartite matching problem and its variations, we refer to Burkard, Dell'Amico and Martello [3]. Another variation of the matching problem is the Minimum Cost Bipartite Perfect Matching Problem with Conflict Pair Constraints (MCBPMPC) which is introduced by Darmann, Pferschy, Schauer and Woeginger [6], in their seminal paper. The authors have shown that the MCBPMPC is strongly $\mathcal{NP}$-hard on a general graph $G$ even if the conflict graph is a collection of single edges. Besides, they have also proved that the MCBPMPC cannot be approximated by a constant factor of the optimal value unless $\mathcal{P} = \mathcal{NP}$.

Given a bipartite graph $G$ with a cost associated with each edge and a conflict set of edge pairs, the MCBPMPC consists of finding a bipartite perfect matching with the lowest total cost such that no more than one edge is selected from each pair in the conflict set. In addition to a bipartite graph $G$ with edge costs, the definition of MCBPMPC uses a conflict set, which consists of edge pairs that are incompatible in a feasible solution. These edge pairs are called conflict pairs and they can alternatively be represented by a conflict graph, in which the nodes correspond to edges of $G$ that are part of the conflict pairs and each edge of $G$ represents a conflict pair. Since the conflict pairs result in binary disjunctive constraints, the MCBPMPC can be used in the applications of matching problems where incompatibilities exist between some pairs of edges. Note that the MCBPMPC arises as a subproblem when solving the Quadratic Bottleneck Assignment Problem (QBAP) which is a generalization of the Bandwidth Minimization Problem in matrices and graphs ([9]).

Öncan, Zhang and Punnen [8] have explored more complexity results for the MCBPMPC and they have shown that when the original graph $G$ is a

[1] This work was supported by the Turkish Scientific and Technological Research Council Research Grants No: 107M462 and 109M139, and Galatasaray University Scientific Research Project Grant No: 16.402.011

[2] Email: `ytoncan@gsu.edu.tr`

[3] Email: `altinel@boun.edu.tr`