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Minimizing the weighted sum of completion times under processing time uncertainty

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Abstract

We address the robust counterpart of a classical single machine scheduling problem by considering a budgeted uncertainty and an ellipsoidal uncertainty. We prove that the problem is $\mathcal{NP}$-hard for arbitrary ellipsoidal uncertainty sets. Then, a mixed-integer linear programming reformulations and a second order cone programming reformulations are provided. We assess the reformulations on randomly generated instances, comparing them with branch-and-cut algorithms.

Keywords: Integer programming, robust optimization, scheduling.

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1 Introduction

Scheduling is a rich topic within combinatorial optimization that has witnessed a large amount of research in the past decades, including applications oriented works, integer programming formulations, polyhedral studies, and approximation algorithms. In this work, we focus in this work on one of the simplest scheduling problem, which can be defined as follows. We are given a set $\mathcal{J} = \{1, \ldots, n\}$ of jobs, each having a processing time p_j and a weight w_j, and we would like to order the jobs so as to minimize the weighted sum of their completion times. Formally, letting $\sigma(i)$ be the position of job i for the permutation σ and $C_j(\sigma) = \sum_{i=1}^{\sigma(j)} p_{\sigma^{-1}(i)}$ be the completion time of job j for permutation σ, we want to solve the optimization problem

$$\min_{\sigma \in P(n)} \sum_{j \in \mathcal{J}} w_j C_j(\sigma), \tag{1}$$

where $P(n)$ represents the set of permutations of $\{1, \ldots, n\}$. It is well-known that Problem (1) can be solved in polynomial time by ordering the jobs according to their non-decreasing value of p_j/w_j, which is known as Smith's rule [12].

In practical scheduling problems, the parameters of the problem are usually subject to variations, and this is particularly true for the vector of processing times p, whose value can be affected by various hazardous events, such as machine breakdowns, working environment changes, worker performance instabilities, to cite a few. We address this issue herein through the lens of min max robust optimization. Specifically, we assume that the uncertainty over p is characterized by a given convex set $U \subset \mathbb{R}^n$, and we study the robust counterpart of (1) that is defined as

$$\min_{\sigma \in P(n)} \max_{p \in U} \sum_{j \in \mathcal{J}} w_j C_j(\sigma, p), \tag{2}$$

where $C_j(\sigma, p) = \sum_{i=1}^{\sigma(j)} p_{\sigma^{-1}(i)}$ denotes the completion time of job j for permutation σ and the vector of processing times taking value p.

For arbitrary uncertainty sets U, it is well known that Problem (2) is $\mathcal{NP}$-hard, see [14], even when $w_j = 1$ for each $j \in \mathcal{J}$ and U is the convex

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