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The Graph Segmentation Problem

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Abstract

We investigate a graph theoretical problem arising in the automatic billing of a network toll. Given a network and a family of user paths, we study the graph segmentation problem (GSP) to cover parts of the user paths by a set of disjoint segments. The GSP is shown to be $\mathcal{NP}$-hard but for special cases it can be solved in polynomial time. We also show that the marginal utility of a segment is bounded. Computational results for real-world instances show that in practice the problem is more amenable than the theoretic bounds suggest.

Keywords: toll billing, graph segmentation, combinatorial optimization

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1 Introduction

In this paper, we introduce the graph segmentation problem (GSP) to cover as many parts as possible of a given set of trajectories (paths in a network) by a set of disjoint segments (which are also paths). The problem has applications in automatic toll billing where the network users have to pay a certain toll for their journey. In such a billing, a user cannot be charged for a specific segment, unless he covers the entire segment during his trip.

An obvious solution to maximize the income is to choose the "atomic" segmentation, i.e., each arc of the network represents a segment. On the other hand, every segment requires specific maintenance, e.g., during construction periods or for manual review of contentious cases. Especially, if we are confronted with a very detailed network but the vast majority of the traffic passes by a great number of nodes, the savings of having fewer segments can be significant. The graph segmentation problem thus maximizes the total income of toll revenues subject to a limited number of segments. Figure 1 depicts optimal solutions of the GSP for the example of the German motorway system.

To the best of our knowledge, no models have been proposed that address this type of graph segmentation problem. The literature on road tolling is rather focused on problems of optimal toll pricing. A classical objective would be to determine arc tolls which will cause minimal congestion in the resulting traffic flow, see e.g. [4,5] for a survey.

The GSP can be formulated as a set packing problem (SPP) of exponential

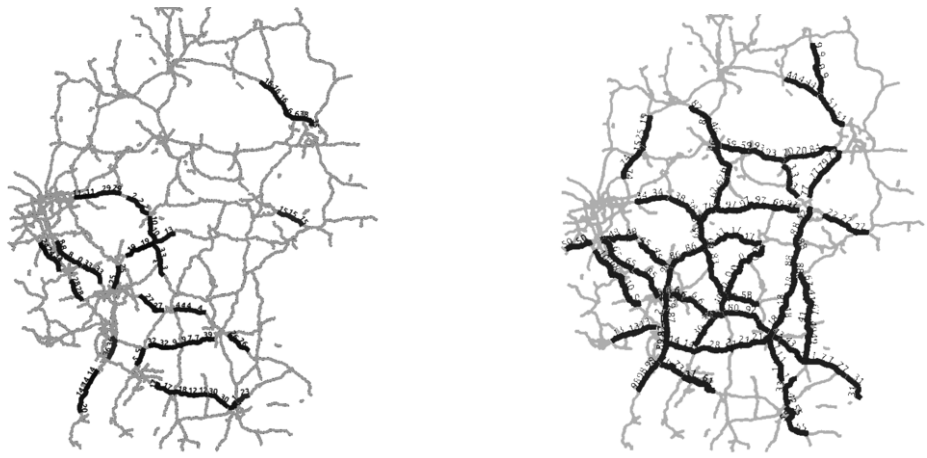

Fig. 1. Optimal segmentation on German motorways with $k = 40$ segments (left) and $k = 100$ segments (right).