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Redundant Coupling/decoupling in Train Unit Scheduling Optimization [1]

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Abstract

A heuristic branch-and-bound approach exploiting flow potentials to reduce coupling/decoupling redundancy in network flow model based train unit scheduling is proposed. We shall first give a proof that if unit types are interchangeable and under certain conditions, fully utilizing an arc will guarantee an improvement. Computational experiments are reported.

Keywords: Train unit scheduling, Coupling/decoupling, Network flow

1 Introduction

Given a fixed timetable on one operational day and a fleet of train units of multiple types, the train unit scheduling problem (TUSP) [4, 5] aims at deriving an optimized plan such that all the trains are covered by train unit(s) with the required seat capacity provisions. From the perspective of a train unit, the problem assigns a sequence of trains to it as its daily workload.

One way to solve the TUSP is to use an integer multicommodity flow (IMCF) model [1], which can be found in [2–5]. Based on a directed acyclic graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, commodities $k \in K$ represent train unit types. $\mathcal{N} = N \cup \{s, t\}$ is the node set where s, t are the source and sink, and N is the

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train node set; $\mathcal{A} = A \cup A_0$ is the arc set where $A = \{(i,j)|i,j \in N\}$ contains connection arcs and $A_0 = \{(s,j)|j \in N\} \cup \{(j,t)|j \in N\}$ contains sign-on/-off arcs. A connection arc $a = (i,j)$ represents a possibility in connecting trains i and j consecutively. A sign-on/-off arc indicates the start/end of the daily work of a train unit, and by default every train node is connected to a sign-on and sign-off arc. An s-t path represents a possible workload for a train unit. Let P^k be the set of paths of type k and x_p be the flow amount in $p \in P^k$. A generic way of modeling the TUSP by IMCF based on paths can be:

$$(P) \quad \min_{x_p} \quad \sum_{k \in K} \sum_{p \in P^k} c_p x_p \tag{1}$$

$$\text{s. t.} \quad \sum_{p \in P^k} x_p \leq b_k, \qquad \forall k \in K; \tag{2}$$

$$\sum_{k \in K_j} \sum_{p \in P_j^k} q_k x_p \geq r_j, \qquad \forall j \in N; \tag{3}$$

$$\sum_{k \in K_j} \sum_{p \in P_j^k} v_k x_p \leq u_j, \qquad \forall j \in N. \tag{4}$$

In (1), x_p is the number of used units along path p and c_p is the relevant cost. (2) requires the used number of units to be within b_k for each type k. (3) asks the capacity provision (q_k is the unit capacity of type k) for each train to satisfy the passenger demand r_j. (4) limits the number of attached cars to be within the maximum required u_j (v_k is the number of cars for type k).

More than one train units can be attached together to serve the same train, which is known as *coupling* (the reverse operation is *decoupling*). The term "coupling/decoupling" will be shortened as "c/d". While appropriate c/d can be important in getting an optimized train unit schedule, if no special control is taken, especially when the schedule is produced by integer multicomodity flow models as in (P), redundant c/d may arise, making the solution impractical in real-world operations.

2 Redundant coupling/decoupling

Let $\#[x]$ and $z(x)$ be the number of c/d operations and the objective value of solution x. The solution over a subgraph $\overline{G} \subset \mathcal{G}$ is denoted as $x(\overline{G})$.

Definition 2.1 In the TUSP, a feasible solution x is said to have redundant c/d operations if there exists at least another feasible solution x' such that