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Vulnerability of Subclasses of Chordal Graphs

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Abstract

In this paper, we introduce a new invariant that supports an accurate evaluation of the connectivity of graphs belonging to some subclasses of chordal graphs, allowing the establishment of a total ordering of the elements of the class. It is based on the minimal vertex separators of the graph, and, as so, its computation is performed in linear time. Results about the behavior of the new invariant and a comparison with the toughness are presented for block graphs.

Keywords: Vulnerability, k-sep chordal graphs, block graphs, minimal vertex separators.

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1 Introduction

Due to several practical applications, different vulnerability measures of graphs have been introduced in the literature over the past few decades. The vertex connectivity is the most studied of these measures; however, it often fails to capture the more subtle properties of distinct structures.

In this paper, we present a new invariant based on the minimal vertex separators of the graph, the *mvs-sequence*. This measure supports a more accurate evaluation of the connectivity of graphs belonging to some subclasses of chordal graphs, allowing to establish a total ordering of the elements of the class. It also contemplates the need to determine the invariant efficiently as its computation is performed in linear time.

A well known theoretical measure of vulnerability is the toughness of the graph, defined by Chvátal [5]. Bauer *et al.* [2] presented a survey about the subject. However, it is well known that, even when dealing with particular classes, it can be hard to compute it efficiently, as it is shown in Section 4. Results about the behavior of the new parameter on block graphs and a comparison with the toughness of these graphs are presented.

2 Basic concepts

Basic concepts about chordal graphs are assumed to be known and can be found in Blair and Peyton [3] and Golumbic [7]. In this section, the most relevant concepts are reviewed.

Let $G = (V, E)$ be a graph, with $|E| = m$, $|V| = n > 0$. The *set of neighbors* of a vertex $v \in V$ is denoted by $N(v) = \{w \in V; \{v, w\} \in E\}$ and its *closed neighborhood* by $N[v] = N(v) \cup \{v\}$. Two vertices u and v are *true twins* in G if $N[u] = N[v]$. A vertex v is said to be *simplicial* in G if $N(v)$ is a clique in G.

A subset $S \subset V$ is a *vertex separator* for non-adjacent vertices u and v (a uv-separator) if the removal of S from the graph separates u and v into distinct connected components. If no proper subset of S is a uv-separator then S is a *minimal uv-separator*. When the pair of vertices remains unspecified, we refer to S as a *minimal vertex separator* (*mvs*).

A *clique-tree* of a chordal graph G is defined as a tree T whose vertices are the maximal cliques of G such that for every two maximal cliques Q and Q' each clique in the path from Q to Q' in T contains $Q \cap Q'$. The set of maximal cliques of G is denoted by $\mathbb{Q}$. Blair and Peyton [3] proved that, for a clique-tree $T = (V_T, E_T)$, a set $S \subset V$ is a minimal vertex separator