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Electronic Notes in DISCRETE MATHEMATICS

Electronic Notes in Discrete Mathematics 64 (2018) 125–134 www.elsevier.com/locate/endm

An exact algorithm for a multicommodity min-cost flow over time problem [1]

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Abstract

Flows over time problems consider finding optimal dynamic flows over a network where capacities and transit times on arcs are given. In this paper we study a multicommodity flow over time problem in which no storage of flow at nodes is allowed

https://doi.org/10.1016/j.endm.2018.01.014

and solutions are restricted to loopless flow-paths. We propose an exact algorithm based on a column generation approach for a path-based linear programming model and present the results of a preliminary computational study.

Keywords: flows over time, multicommodity flow, linear programming, column generation, exact algorithm.

1 Introduction

In most network flows models, time dimension is not explicitly considered. In fact, in several applications such as road and air traffic management, or water distribution this assumption is unrealistic. *Flow over time* (or dynamic flows) models overcome these shortcomings as the flow requires a positive amount of time to travel through an arc and, in addition, it is allowed to vary over time. The notion of flow over time was introduced by Ford and Fulkerson in [5] where efficient solution algorithms are presented for the *maximum flow over time* problem: Given a capacitated network, origin and destination nodes, and arcs transit times, find a flow over time maximizing the amount of flow reaching the destination node within a given time horizon T. Capacity constraints are expressed as upper bounds on the flow rates on the arcs. An introduction to the area of flows over time can be found in [11], where many results obtained over the last fifty years are summarized.

Klinz and Woeginger [10] show that the single origin, single destination *minimum cost flow over time* is binary NP-hard even for series-parallel graphs. For the same setting, Fleischer and Skutella show that storage of flow at intermediate nodes (which implies that, at each node, the starting time of the commodity can be decided) is never beneficial [3]. The *multicommodity* setting is, of course, NP-hard as well, as showed by Hall, Hippler and Skutella [8]. The authors prove that, when arc costs are all null, finding a feasible flow for $k \geq 2$ commodities within a given time horizon, is binary NP-complete and it can be found in pseudopolynomial time on the so-called Time Expanded Network (hereafter, TEN). Several flow over time problems may be reduced to their static counterparts by using a TEN which, roughly speaking, introduces copies

[1] This work has been partially supported by Italian MIUR projects PRIN-COFIN n. 2012JXB3YF_004 and 2012C4E3KT_001

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