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Electronic Notes in DISCRETE MATHEMATICS

Electronic Notes in Discrete Mathematics 64 (2018) 145–154 www.elsevier.com/locate/endm

Formulation and algorithms for the robust maximal covering location problem

Amadeu A. Coco[a,b,2], Andréa Cynthia Santos[b,4], Thiago F. Noronha[a,3]

[a] *DCC, Universidade Federal de Minas Gerais, Belo Horizonte, Brazil*

[b] *ICD-LOSI, Université de Technologie de Troyes, Troyes, France.*

Abstract

Let N be the line-set and M be the column-set of a matrix $\{a_{ij}\}$, such that $a_{ij} = 1$ if line $i \in N$ is covered by column $j \in M$, or $a_{ij} = 0$ otherwise. Besides, let $b_j \geq 0$ be the benefit associated with a column $j \in M$. Given a constant $T < |M|$, the NP-Hard Maximal Covering Location Problem (MCLP) consists in finding a subset $X \subseteq M$ with the maximum sum of benefits, such that $|X| \leq T$ and every line in N is covered by at least one column in X. In this study, we investigate the min-max regret Maximal Covering Location Problem, a robust counterpart of MCLP, where the benefit of each column is uncertain and modeled as an interval data. The objective is to find a robust solution that minimizes the maximal regret over all possible combinations of values for the columns benefit. This problem has applications in disaster relief. We propose a MILP formulation, an exact and 2-approximation algorithms, and test them using classical instances from the literature.

Keywords: Robust optimization, min-max regret, uncertain data, heuristics.

[1] This work was partially supported by CNPq, CAPES, and FAPEMIG

[2] Email: amadeuac@dcc.ufmg.br

[3] Email: tfn@dcc.ufmg.br

[4] Email: andrea.duhamel@utt.fr

https://doi.org/10.1016/j.endm.2018.01.016

1 Introduction

Robust optimization is an approach to deal with data uncertainty where the variability of the data is represented by deterministic values [2,11,14]. In this work, we focus on robust optimization approaches where uncertain parameters are modeled as an interval of possible values. We refer to the book [14] and the survey [2] for other robust optimization approaches. Many robust counterparts of classical optimization problems have been studied in the literature, such as the Robust Shortest Path Problem [7,10], the Robust Minimum Spanning Tree Problem [19] and the Robust Assignment Problem [17]. These problems are NP-Hard, despite the fact that their classical counterparts are solved in polynomial time. Recently, significant attention has been given to the robust counterpart of NP-Hard problems, such as the Robust Traveling Salesman Problem [16], the Robust Set Covering Problem [18], the Robust Knapsack Problem [9] and the Robust Restricted Shortest Path Problem [3].

We deal with a generalization of the Maximal Covering Location Problem (MCLP), which was introduced in [6]. Let $\{a_{ij}\}$ be a matrix with a set N of lines and a set M of columns, where each column $j \in M$ is associated with a benefit $b_j \geq 0$. Given a constant $T < |M|$, MCLP consists in finding a subset $X \subseteq M$ with the maximum benefit sum, such that $|X| \leq T$ and every line in N is covered by at least one column in X. Practical applications of MCLP are described in [15]. This problem is NP-hard, and exact and approximation algorithms were proposed in [1,6].

In this study, we introduce the min-max regret Maximal Covering Location Problem (MMR-MCLP), a robust counterpart of MCLP, where the benefit of each column is uncertain and modeled as an interval of possible values. Let N, M, $\{a_{ij}\}$ and T be as defined above, and $[l_j, u_j]$ be an interval with the minimum and the maximum benefit expected for column $j \in M$. We define a scenario $s \in S$ as an assignment of a single value $b_j^s \in [l_j, u_j]$ for each column $j \in M$, where S is the set of all possible values for the columns benefit. As MCLP, MMR-MCLP consists in finding $X \subseteq M$, such that $|X| \leq T$ and every line in N is covered by at least one column in X, and one looks for a robust solution that minimizes the maximal regret over all scenarios.

Let S be the set of scenarios, and Γ be the set of feasible solutions. Let also $\psi^s(X) = \sum_{j \in X} b_j^s$ be the benefit of the solution $X \in \Gamma$ for the scenario $s \in S$, where b_j^s is the benefit of column $j \in M$ in s. The *regret* of a solution $X \in \Gamma$ for a scenario $s \in S$ is defined as the difference between $\psi^s(X^s)$ and $\psi^s(X)$, where X^s is the optimal solution for the scenario s, i.e. the regret of using X instead of X^s if scenario s occurs. MMR-MCLP aims at finding the