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# Compact and Extended Formulations for the Virtual Network Embedding Problem

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**Abstract**

Virtualization allows multiple virtual networks to be nested in a common substrate network sharing resources such as CPU, memory and bandwidth. For that, a Virtual Network Embedding problem needs to be solved, allocating virtual requests to physical components with available capacities. This problem is NP-hard and exact approaches still present many challenges to be tackled. In this work three formulations are compared showing the strengths of the linear programming relaxations. For that, column generation algorithms are devised, indicating optimality may be achieved faster through decomposition approaches.

*Keywords:* Combinatorial optimization, integer programming, column generation, formulations, virtual network embedding.

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# 1 Introduction

The Virtual Network Embedding Problem (VNE) aims to map a set of virtual network requests onto a physical substrate network ensuring physical capacities are not exceeded. The substrate network is given by a weighted undirected graph $G^0 = (N^0, E^0)$ with a set of physical nodes ($N^0$) and a set of physical links ($E^0$), where each node $i \in N^0$ is associated to a CPU capacity $C_i$ while each link $\{i, j\} \in E^0$ has a bandwidth capacity $B_{ij}$. Similarly, each virtual request $v \in V$ is given by a weighted undirected graph $G^v = (N^v, E^v)$, where a virtual node $k \in N^v$ has a CPU demand $c^v_k$ and a virtual link $\{k, l\} \in E^v$ has a bandwidth demand $b^v_{kl}$, and is associated to a profit $P^v$. A VNE solution consists of attending a subset $V' \subseteq V$ which maximizes the total profit [4]. This problem is proven to be strongly NP-hard by [1].

Network virtualization has been used to circumvent the *ossification of the internet* allowing the implementation and run of new features [5]. This phenomenon is assigned to the barriers of IPv4, high expansion costs, difficulties for advancing the internet of things, among others. Although not very new, virtualization is a promising technique to overcome the resistance of the current internet to fundamental changes [2]. Virtualization technology has been adopted increasingly in order to improve efficiency and agility of computing and storage in software-based resources. Some of the main advantages of virtualization include flexibility, scalability, isolation, cost reduction and safety.

Virtualization presents several challenges to overcome, bringing out problems such as VNE. Besides, characteristics and assumptions closer to the real scenarios should be incorporated into models and algorithms. Most of literature solutions are devoted to heuristic approaches, aiming to deal with real world dimensions and the online nature inherent in the problem. However, exact approaches seems interesting not only due to its theoretical importance but also because advanced techniques are allowing larger problems to be tackled. Three integer programming formulations are explored in this work, aiming to compare bounds and algorithm perspectives. A compact flow-based and two extended formulations based on paths and configurations are presented.

The remaining of the paper is organized as follows. In section 2, literature works are briefly reviewed. Integer programming formulations are introduced in Section 3 while algorithms are detailed in Section 4. Preliminary results and discussion are presented in Section 5. Finally, Section 6 concludes the work and gives future directions of the research.

[4] Other objective functions can be applied to VNE. The maximization of profit is used in this work.