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Compact Models for Critical Node Detection in Telecommunication Networks

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Abstract

Given a network defined by a graph, a weight associated to each node pair and a positive parameter p, the CND problem addressed here is to identify a set of at most p critical nodes minimizing the total weight of the node pairs that remain connected when all critical nodes are removed. We improve previously known compact models and present computational results, based on telecommunication backbone networks, showing that the proposed models are much more efficiently solved and enable us to obtain optimal solutions for networks up to 200 nodes and p values up to 20 critical nodes within a few minutes in the worst cases.

Keywords: Critical Node Detection, Network Vulnerability, Mixed Integer Linear Programming, Optimization, Telecommunication Networks

1 Introduction

Critical Node Detection (CND) problems aim to identify a set of optimal nodes (the critical nodes) on a given network that, if removed, minimize or restrict a given metric of network connectivity. CND can be defined either by imposing

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an upper bound on the number of critical nodes and minimizing a connectivity metric, or by imposing a connectivity metric lower bound and minimizing the number of critical nodes. In [1], CND is defined as the detection of a given number of critical nodes aiming to minimize the pairwise connectivity. It shows that the problem is NP-hard and proposes both a compact model and a multi-start local search heuristic algorithm. A more compact model is proposed in [7] for the same problem, together with reformulations and valid inequalities, while a path based model with a non-polynomial number of constraints is proposed in [2]. A recent work [8] considers nodes with associated costs and the aim is to identify a set of critical nodes, within a given cost budget, whose removal minimizes a distance-based connectivity metric. The aim in [3] is to identify a minimum set of critical network elements, referred to as a $\beta-$disruptor, whose removal results in a specific pairwise connectivity target ($0 \leq \beta < 1$ denotes the connectivity fraction target). Network elements can be either links or nodes. More recently, [4] assumes a given set of link and node costs and extends the work in [3] to the case where the $\beta-$disruptor can be a mix of links and nodes.

Given the rising risk of natural (hurricanes, tsunamis, or earthquakes) and human (security attacks) causes, the impact of large-scale failures in telecommunication networks is becoming a hot topic since such failures can seriously disrupt network services [6]. A key component when dealing with these issues is the evaluation of current networks' vulnerability to large-scale failures. The CND problem, which has been studied in general contexts (social networks, power grids, military networks, biology, and so on), is gaining special interest in the vulnerability evaluation of telecommunication networks [5]. Such networks, especially in telecommunication backbone scenarios, are quite different from the networks addressed in previous works, since they are strongly connected in the sense that their topologies provide at least two node disjoint paths between any pair of nodes, which makes CND more challenging.

The CND variant addressed in this work differs from the one in [1,7] only in the objective function. We associate a weight to each pair of network nodes (representing the importance of the traffic between each node pair) and the aim is the minimization of the total weight of the node pairs that remain connected (in [1,7], the weight values are all one). In this paper, we start by considering the models proposed in [1,7] and reformulate both of them to derive new mixed integer linear programming models. We present computational results based on telecommunication backbone network instances showing that the new models are much more efficiently solved and enable us to obtain optimal solutions for networks up to 200 nodes and for p values up to 20.