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On Edge-Graceful Regular Graphs with Particular 3-Factors

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Abstract

An edge-graceful labeling of a finite simple graph with p vertices and q edges is a bijection from the set of edges to the set of integers $\{1, 2, \cdots, q\}$ such that the vertex sums are pairwise distinct modulo p, where the vertex sum at a vertex is the sum of labels of all edges incident to such vertex. A graph is called edge-graceful if it admits an edge-graceful labeling. In this article, we verify that an regular graph of odd degree is edge-graceful if it contains either of two particular 3-regular spanning subgraphs, namely, a quasi-prism factor and a claw factor.

Keywords: edge-graceful labeling, regular graph, factor, claw factor, quasi-prism factor.

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1 Introduction and Background

All graphs in this paper are finite simple, undirected, and without loops unless otherwise stated. In 1990, N. Hartsfield and G. Ringel [4] introduced the concepts called antimagic labeling and antimagic graphs.

Definition 1.1 *Let $G = (V, E)$ be a graph with p vertices, q edges, and without any isolated vertex. An* **antimagic** *edge labeling is a bijection $f : E \to \{1, 2, \cdots, q\}$, such that the induced vertex sum $f^+ : V \to \mathbb{Z}^+$ given by $f^+(u) = \sum\{f(uv) : uv \in E\}$ is injective. A graph is called* **antimagic** *if it admits an antimagic labeling. If moreover for G the vertex sums f^+ are consecutive integers, we say G admits an* $(a, 1)$**-antimagic** *labeling and G is* $(a, 1)$**-antimagic**.

Definition 1.2 *Let $G = (V, E)$ be a graph with p vertices, q edges, and without any isolated vertex. An* **edge-graceful** *edge labeling is a bijection $f : E \to \{1, 2, \cdots, q\}$, such that the induced vertex sum $f^+ : V \to \mathbb{Z}_p$ given by $f^+(u) = \sum\{f(uv) : uv \in E\}$ $(mod\ p)$ is injective. A graph is called* **edge-graceful** *if it admits an edge-graceful labeling.*

Note that an $(a, 1)$-antimagic labeling is an edge-graceful labeling, and an edge-graceful labeling is necessarily an antimagic labeling.

In 1985 S.P. Lo [6] introduced such notion edge-graceful labeling. In 2005 D. Hefetz [5] proved that, for an edge-graceful graph G, it is still edge-graceful after adding an arbitrary even factor. In 2008, T.-M. Wang [8] studied edge-graceful spectrum of graphs. Most recently Bača, Semaničová-Feňovčíková and the present authors [2] studied the existence for $(a, 1)$-antimagic-ness of certain 3-regular graphs. Many various types of graphs have been shown to be antimagic [2,5,8,9,10] over the years. For more conjectures and open problems on various types of antimagic labeling problems, interested readers are recommended to refer to the dynamic survey article of J. Gallian [3].

In this paper, we show that an odd regular graph is edge-graceful if it contains a quasi-prism factor or a claw factor.

2 Odd Regular Graphs with a Quasi-Prism Factor

The following is a necessary condition for being an edge-graceful graph:

Lemma 2.1 *Let G be a graph with p vertices and q edges. If G is edge-graceful, then $q(q + 1) \equiv \frac{p(p-1)}{2}$ $(mod\ p)$.*