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Genus of total graphs of commutative rings : A survey

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Abstract

The topological properties of graphs are well studied. More specifically the genus of graphs and their embeddings are attractions to many authors. In this survey, we present the results concerning the genus of total graph and generalized the total graph and generalized total graphs of commutative rings.

Keywords: Commutative ring, total graph, Cayley graph, planar, toroidal, genus.

1 Introduction

The study of algebraic structures, using tools of graph theory, tends to an exciting topic of research during the last two decades. There are so many ways to associate a graph with an algebraic structure. Some of them to mention are Cayley graphs from groups, non-commutating graph of a group, power graph of a finite group, zero-divisor graph of a ring, total graph of a ring, unit graph

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of a ring, co-maximal graph of a ring, annihilating ideal graph of a ring and torsion graph over modules. The study about these graphs constructed out of rings deal with interplay between the algebraic properties of a ring and the graph theoretical properties of the corresponding graph.

The idea of associating a graph with zero-divisors of a commutative ring was introduced by I. Beck [7] in 1988. In 1999, D. F. Anderson and P. S. Livingston modified the definition of I. Beck. There after, many research articles have been published on the zero-divisor graph of commutative rings. In variation to the concept of zero- divisor graph, D. F. Anderson and A. Badawi [3] introduced the total graph of a commutative ring. The *total graph* of R, denoted by $T_\Gamma(R)$, is the undirected graph whose vertices are the elements in R and two distinct vertices x and y are adjacent if $x + y \in Z(R)$. In recent years, many research articles have been published on total graph of commutative rings [1,3,10,12,13].

Akhtar et al. [2] defined the *unitary Cayley graph* of R, denoted by $Cay(R, U(R))$, as the graph whose vertex set is R and two distinct vertices x and y are adjacent if $x - y \in U(R)$. Khashyarmanesh et al. [8] introduced the graph $\Gamma(R, G, S)$ where G is a multiplicative subgroup of $U(R)$ and S is a non-empty subset of G such that $S^{-1} = \{s^{-1} : s \in S\} \subseteq S$. The graph $\Gamma(R, G, S)$ is a simple graph with vertex set R and two distinct vertices x and y are adjacent if there exists $s \in S$ such that $x + sy \in G$.

Note that all these graphs are not isomorphic. But there are relations between these graphs. For instance, if $S = \{1\}$, then $\Gamma(R, U(R), S)$ is same as the unit graph $G(R)$ and if $S = \{-1\}$, then $\Gamma(R, U(R), S)$ is the unitary Cayley graph $Cay(R, U(R))$ of R. Further if R is finite, then the complement $\overline{T_\Gamma(R)}$ of the total graph $T(\Gamma(R))$ is nothing but the unit graph $G(R)$. In case of an infinite ring, $G(R)$ is a subgraph of $\overline{T_\Gamma(R)}$.

For non-negative integers g and k, let S_g denote the sphere with g handles and N_k denote a sphere with k crosscaps attached to it. It is well-known that every connected compact surface is homeomorphic to S_g or N_k for some non-negative integers g and k. The genus of a graph G, denoted by $g(G)$, is the minimum integer n such that the graph can embedded in S_n. Similarly, *crosscap number* (nonorientable genus) $\tilde{g}(G)$ is the minimum k such that G can be embedded in N_k. A graph G is called a *planar* if $g(\Gamma(G)) = 0$ (i.e., the graph's vertices can be placed on a plane such that no edges cross), and *toroidal* if $g(\Gamma(G)) = 1$ (i.e., the graph's vertices can be placed on a torus such that no edges cross).

Moreover, a nonorientable genus 1 graph is called a *projective graph.* Note that if H is a subgraph of a graph G, then $g(H) \le g(G)$ and $\tilde{g}(H) \le \tilde{g}(G)$.