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Electronic Notes in DISCRETE MATHEMATICS

Electronic Notes in Discrete Mathematics 63 (2017) 69–76

www.elsevier.com/locate/endm

Reduced power graph of a group

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Abstract

Let S be a semigroup. We define the *directed reduced power graph of* S, denoted by $\overrightarrow{\mathscr{P}}(S)$, is a digraph with vertex set S, and for $u, v \in S$, there is an arc from u to v if and only if $u \neq v$ and $\langle v \rangle \subset \langle u \rangle$. The *(undirected) reduced power graph of* S, denoted by $\mathscr{P}(S)$, is the underlying graph of $\overrightarrow{\mathscr{P}}(S)$. This means that the set of vertices of $\mathscr{P}(S)$ is equal to S and two vertices u and v are adjacent if and only if $u \neq v$ and $\langle v \rangle \subset \langle u \rangle$ or $\langle u \rangle \subset \langle v \rangle$. In this paper, we study some interplay between the algebraic properties of a group and the graph theoretic properties of its (directed and undirected) reduced power graphs. Also we establish some relationship between the reduced power graphs and power graphs of groups.

Keywords: Group, Reduced power graph, Power graph.

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[3] This research work of the second author is supported by Ministry of Social Justice & Empowerment and Ministry of Tribal Affairs, India in the form of Rajiv Gandhi National Fellowship.

https://doi.org/10.1016/j.endm.2017.10.063

1 Introduction

The study of the properties of algebraic structures by the graph theoretic tools has been a main approach of algebraic graph theory. In recent years, the power graphs associated with groups and semigroups have been investigated by several researchers.

In [5], Kelarev and Quinn introduced and studied the directed power graph of a semigroup. The *directed power graph* of a semigroup S, denoted by $\overrightarrow{\mathcal{P}}(S)$, is a digraph with vertex set S, and for $u, v \in S$, there is an arc from u to v if and only if $u \neq v$ and $v = u^n$, for some positive integer n; which is equivalent to say $u \neq v$ and $\langle v \rangle \subseteq \langle u \rangle$. Following this, in [3], Chakrabarty et al. defined the *undirected power graph* of S, denoted by $\mathcal{P}(S)$, which is an undirected graph whose vertex set is S, and two vertices u and v are adjacent if and only if $u \neq v$ and $u^n = v$ or $v^n = u$ for some positive integer n; which is equivalent to say $u \neq v$ and $\langle v \rangle \subseteq \langle u \rangle$ or $\langle u \rangle \subseteq \langle v \rangle$. Given a group G, the subgraph of $\overrightarrow{\mathcal{P}}(G)$ (resp. $\mathcal{P}(G)$) induced on $G \setminus \{e\}$ is denoted by $\overrightarrow{\mathcal{P}}^*(G)$ (resp. $\mathcal{P}^*(G)$), where e denotes the identity element of G. We refer the reader to the survey [4] for further results and open problems on the (directed and undirected) power graphs of groups and semigroups.

In this paper, we define the following graphs: Let S be a semigroup. The *directed reduced power graph of* S, denoted by $\overrightarrow{\mathscr{P}}(S)$, is a digraph with vertex set S, and for $u, v \in S$, there is an arc from u to v if and only if $u \neq v$, $v = u^n$ for some positive integer n and $\langle v \rangle \neq \langle u \rangle$; which is equivalent to say $u \neq v$ and $\langle v \rangle \subset \langle u \rangle$. The *(undirected) reduced power graph of* S, denoted by $\mathscr{P}(S)$, is the underlying graph of $\overrightarrow{\mathscr{P}}(S)$. This means that the set of vertices of $\mathscr{P}(S)$ is equal to S and two vertices u and v are adjacent if and only if $u \neq v$, $u^n = v$ and $\langle v \rangle \neq \langle u \rangle$ or $v^n = u$ and $\langle v \rangle \neq \langle u \rangle$ for some positive integer n; which is equivalent to say $u \neq v$ and $\langle v \rangle \subset \langle u \rangle$ or $\langle u \rangle \subset \langle v \rangle$. Given a group G, we denote the subgraph of $\overrightarrow{\mathscr{P}}(G)$ (resp. $\mathscr{P}(G)$) induced on $G \setminus \{e\}$ by $\overrightarrow{\mathscr{P}}^*(G)$ (resp. $\mathscr{P}^*(G)$), where e denotes the identity element of G.

Clearly $\overrightarrow{\mathscr{P}}(G)$ is a spanning subdigraph of $\overrightarrow{\mathcal{P}}(G)$, and $\mathscr{P}(G)$ is a spanning subgraph of $\mathcal{P}(G)$.

The structure of $\mathcal{P}(\mathbb{Z}_{10})$, $\mathscr{P}(\mathbb{Z}_{10})$ and $\mathscr{P}^*(\mathbb{Z}_{10})$ are described in Figures 1(a), 1(b) and 1(c), respectively.

We use the standard notations and terminologies of graph theory following [1]. In a simple graph G, the degree of a vertex v in G is denoted by $deg_G(v)$, and $gr(G)$ denote the girth of G. K_n denote the complete graph on n vertices. P_n and C_n denotes the path and the cycle of length n, respec-