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Complexity Issues of Variants of Secure Domination in Graphs

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Abstract

Let $G = (V, E)$ be a simple, undirected and connected graph. A connected dominating set $S \subseteq V(G)$ is a secure connected dominating set of G, if for each $u \in V(G) \setminus S$, there exists $v \in S$ such that $(u, v) \in E(G)$ and the set $(S \setminus \{v\}) \cup \{u\}$ is a connected dominating set of G. The minimum cardinality of a secure connected dominating set of G denoted $\gamma_{sc}(G)$, is called the secure connected domination number of G. In this paper, we show that the decision problems of finding a minimum secure connected dominating set and a minimum secure total dominating set are NP-complete for split graphs. We initiate the study of 2-*secure domination* and show that the decision problem corresponding to the computation of 2-secure domination number of a graph is NP-complete, even when restricted to split graphs or bipartite graphs. Finally we show that given two positive integers k (≥ 2) and n $(\geq max\{4, k\})$ there exists a graph G with $|V(G)| = n$ and $\gamma_{sc}(G) = k$.

Keywords: Secure dominating set, split graph, NP-complete, total domination.

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1 Introduction

Let $G(V, E)$ be a simple, undirected and connected graph. We use commonly used terminology and notations as found in popularly available texts such as [2,7]. For a vertex $v \in V(G)$, the (*open*) *neighborhood* of v in G is $N(v)=\{u \in V(G) : (u, v) \in E(G)\}$, the *closed neighborhood* of v is defined as $N[v] = N(v) \cup \{v\}$. If $S \subseteq V(G)$, then the (open) neighborhood of S is the set $N(S) = \cup_{v\in S} N(v)$. The closed neighborhood of S is $N[S] = S \cup N(S)$. Let $S \subseteq V(G)$ and v be a vertex in S. The S-*external private neighborhood* of v denoted $epn[v, S]$ is defined as $\{w \in V(G)|N(w) \cap S = \{v\}\}$.
A subset S of $V(G)$ is a *dominating set* (DS) in G if for every $u \in V(G) \setminus S$, there exists $v \in S$ such that $(u, v) \in E(G)$, i.e., $N[S] = V(G)$. The *domination number* of G is the minimum cardinality of a DS in G and is denoted by $\gamma(G)$. A set $S \subseteq V(G)$ is said to be a *secure dominating set* (SDS) in G if for every $u \in V(G) \setminus S$ there exists $v \in S$ such that $(u, v) \in E(G)$ and $(S \setminus \{v\}) \cup \{u\}$ is a dominating set. The minimum cardinality of a SDS in G is called the *secure domination number* of G and is denoted by $\gamma_s(G)$. A dominating set S is said to be a *connected dominating set* (CDS), if the induced subgraph $G[S]$ is connected. Let S be a CDS in G. A CDS S is called a *secure connected dominating set* (SCDS) in G, if for each $u \in V(G) \setminus S$, there exists $v \in S$ such that $(u, v) \in E(G)$ and $(S \setminus \{v\}) \cup \{u\}$ is a CDS in G. The *secure connected domination number* $\gamma_{sc}(G)$ of G is the smallest cardinality of a SCDS in G. A dominating set S is said to be a *total dominating set* (TDS), if the induced subgraph $G[S]$ has no isolated vertices. A TDS S is called a *secure total dominating set* (STDS) of G, if for each $u \in V(G) \setminus S$, there exists $v \in S$ such that $(u, v) \in E(G)$ and $(S \setminus \{v\}) \cup \{u\}$ is a TDS in G. The *secure total domination number* $\gamma_{sc}(G)$ of G is the smallest cardinality of a STDS in G.

2 Complexity of Secure Domination Parameters

In this section we determine the algorithmic complexity of different secure domination parameters when restricted to split or bipartite graphs.

2.1 Algorithmic complexity of secure connected (total) domination in split graphs

First we recall the following definition of *split* graph.

Definition 2.1 A graph is a *split* if its vertex set is the disjoint union of a clique and independent set.