Available online at www.sciencedirect.com

ScienceDirect

Electronic Notes in DISCRETE MATHEMATICS

Electronic Notes in Discrete Mathematics 63 (2017) 85–91

www.elsevier.com/locate/endm

Further results on the radio number of trees

Devsi Bantva[1]

Department of Mathematics
Lukhdhirji Engineering College, Morvi - 363 642, Gujarat, India

Abstract

Let G be a finite, connected, undirected graph with diameter $\mathrm{diam}(G)$ and $d(u,v)$ denote the distance between u and v in G. A radio labeling of a graph G is a mapping $f : V(G) \to \{0, 1, 2, ...\}$ such that $|f(u) - f(v)| \geq \mathrm{diam}(G) + 1 - d(u,v)$ for every pair of distinct vertices u, v of G. The radio number of G, denoted by $\mathrm{rn}(G)$, is the smallest integer k such that G has a radio labeling f with $\max\{f(v) : v \in V(G)\} = k$. In this paper, we determine the radio number for three families of trees obtained by taking graph operation on a given tree or a family of trees.

Keywords: Radio labeling, radio number, tree.

1 Introduction

A number of graph labelings have a root in channel assignment problem. In the channel assignment problem, we seek to assign channels to transmitters such that it satisfies all interference constraints. This well explored problem is also studied using graph coloring. In a graph, a set of transmitters is represented by vertices of a graph; two vertices are adjacent if transmitters are *very close* and at distance two apart if transmitters are *close* in a network. Notice that two transmitters are classified as *very close* if the interference is unavoidable

[1] Email:`devsi.bantva@gmail.com` (Devsi Bantva)

https://doi.org/10.1016/j.endm.2017.11.002

and *close* if the interference is avoidable between them. Motivated by this problem Griggs and Yeh [9] introduced the following $L(2,1)$-labeling problem: An $L(2,1)$-*labeling* (or *distance-two labeling*) of a graph $G = (V(G), E(G))$ is a function f from the vertex set $V(G)$ to the set of non-negative integers such that $|f(u) - f(v)| \geq 2$ if $d(u,v) = 1$ and $|f(u) - f(v)| \geq 1$ if $d(u,v) = 2$. The *span* of f is defined as $\max\{|f(u) - f(v)| : u, v \in V(G)\}$, and the minimum span over all $L(2,1)$-labelings of G is called the λ-*number* of G, denoted by $\lambda(G)$. Observe that $L(2,1)$-labeling deal with two level interference only. The $L(2,1)$-labeling and other distance-two labeling problems have been studied by many researchers in the past two decades; see [5] and [20].

In 2005, Chartrand *et al.* [7] introduced the concept of radio labeling and put the level of interference at largest possible-the diameter of graph. Denote by $\mathrm{diam}(G)$ the *diameter* of G, that is, the maximum distance among all pairs of vertices in G.

Definition 1.1 A *radio labeling* of a graph G is a mapping $f : V(G) \to \{0, 1, 2, \ldots\}$ such that for every pair of distinct vertices u, v of G,

$$d(u,v) + |f(u) - f(v)| \geq \mathrm{diam}(G) + 1.$$

The integer $f(u)$ is called the *label* of u under f, and the *span* of f is defined as $\mathrm{span}(f) = \max\{|f(u) - f(v)| : u, v \in V(G)\}$. The *radio number* of G is defined as

$$\mathrm{rn}(G) := \min_f \mathrm{span}(f)$$

with minimum taken over all radio labelings f of G. A radio labeling f of G is *optimal* if $\mathrm{span}(f) = \mathrm{rn}(G)$.

Note that any optimal radio labeling must assign 0 to some vertex and also in the case when $\mathrm{diam}(G) = 2$ we have $\mathrm{rn}(G) = \lambda(G)$. Observe that any radio labeling should assign different labels to distinct vertices. In fact, a radio labeling induces an ordering u_0, u_1 ,...,u_{p-1} ($p = |V(G)|$) of vertices such that $0 = f(u_0) < f(u_1) < ... < f(u_{p-1}) = \mathrm{span}(f)$.

The radio number of graphs is studied by limited group of authors. The readers are advised to refer the following papers for the radio number of listed graph families; [7,8,15,21] for paths and cycles, [13,14] for square of paths and cycles, [3] for all graphs of order n and diameter $n-2$, [4] for distance graphs, [1,2,12] about radio number of trees, [11] for complete m-ary trees, [10] for level-wise regular trees, [16] for total graph of paths, [17] for strong product $P_2 \boxtimes P_n$, [18] for linear cacti. One can also refer to the survey article [6] for detail on the radio number of graphs.