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On Sum of Powers of the Laplacian Eigenvalues of Power Graphs of Certain Finite Groups

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Abstract

The power graph $\mathcal{G}(G)$ of a finite group G is the graph whose vertices are the elements of G and two distinct vertices are adjacent if and only if one is an integral power of the other. Here we concentrate on sum of powers of the non-zero Laplacian eigenvalues of $\mathcal{G}(\mathbb{Z}_n)$ and $\mathcal{G}(D_n)$. As a result we obtain bounds for Laplacian-energy-like (LEL) invariant of the same graphs.

Keywords: Power Graphs, Finite Groups, Laplacian spectrum.

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1 Introduction

Directed power graph of a semigroup S was introduced by Kelarev and Quinn [7]. This digraph has vertex set S and for $x, y \in S$ there is an arc from x to y if and only if $x \neq y$ and $y = x^m$ for some positive integer m. After that Chakrabarty et al. [3] defined the *power graph* $\mathcal{G}(S)$ of a semigroup S as an undirected graph whose vertex set is S and two vertices $u, v \in S$ are adjacent if and only if $u \neq v$ and $u^m = v$ or $v^m = u$ for some positive integer m. In [3] it was shown that the power graph of any finite group G is connected and the number of edges e of the power graph of a finite cyclic group of order n is $\frac{1}{2}\sum_{d|n}\{2d - \phi(d) - 1\}\phi(d)$. For more properties of power graphs of finite groups like isomorphism, vertex-connectivity etc. we refer [2,4]. We encourage the reader to see the paper [1] for an extensive survey on power graphs.

Recall that for any finite simple undirected graph G the Laplacian matrix $L(G)$ is given by $L(G) = D(G) - A(G)$, where $A(G)$ is the adjacency matrix of G and $D(G)$ is the diagonal matrix of vertex degrees. In the paper [5] we have studied Laplacian spectrum of the graphs $\mathcal{G}(\mathbb{Z}_n)$ and $\mathcal{G}(D_n)$ where $\mathbb{Z}_n$ is the additive group of integers modulo n and D_n denotes the dihedral group of degree n.

For a nonzero real number α, let $s_\alpha(G)$ be the sum of α^{th} power of the nonzero Laplacian eigenvalues of the graph G. In [11] Zhou established bounds for $s_\alpha(G)$ for general and bipartite graphs. It is known that the sum of the Laplacian eigenvalues of a graph G, which is $s_1(G)$, is equal to $2M$, where M is the number of edges of G. So $\alpha = 1$ is a trivial case. On the other hand $s_{\frac{1}{2}}(G)$ is known as Laplacian-energy-like invariant (LEL for short) of the graph G, which was first introduced and studied by Liu and Liu [8]. After that many researchers have found bounds of LEL for general graphs as well as special graphs like trees, star graphs, paths etc., for instance, see [6,10]. In [9], it was shown that LEL describes the properties like entropy, molar volume, molar refraction and also more difficult properties like boiling point, melting ponit and partition coefficient.

In this paper we improve lower bound of $s_\alpha(G)$ for $\alpha < 0$ or $\alpha > 1$ and upper bound of $s_\alpha(G)$ for $0 < \alpha < 1$ given by Zhou [11] for the particular class of graphs $\mathcal{G}(\mathbb{Z}_n)$ and $\mathcal{G}(D_n)$. Moreover we found lower bounds of $s_\alpha(\mathcal{G}(\mathbb{Z}_n))$ and $s_\alpha(\mathcal{G}(D_n))$ for $0 < \alpha < 1$ in terms of number of vertices and Zagerb index. As a result we get bounds for Laplacian-energy-like invariant of these graphs.