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Degree exponent polynomial of graphs obtained by some graph operations

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Dedicated to the memory of Late Dr. B. D. Acharya

Abstract

The degree exponent matrix $DE(G)$ of a graph G is a square matrix whose (i, j)-th entry is $d_i^{d_j}$ whenever $i \neq j$, otherwise it is zero, where d_i is the degree of the i-th vertex of G. In this paper we obtain the characteristic polynomial of the degree exponent matrix of graphs obtained by some graph operations.

Keywords: Degree exponent matrix, Degree exponent polynomial, Eigenvalues, Graph operations.

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1 Introduction

There are several graph polynomials based on the matrices such as adjacency matrix [5], Laplacian matrix [9], signless Laplacian matrix [6,10], distance matrix [1], degree sum matrix [8,11], Seidel matrix [2]. The purpose of this paper is to study the characteristic polynomial of the new matrix of a graph, called degree exponent matrix.

Let G be a simple, undirected graph with n vertices and m edges. Let $V(G)$ be the vertex set and $E(G)$ be the edge set of G. The *degree* of a vertex v in G is the number of edges incident to it and is denoted by $deg_G(v)$. If the degree of each vertex of G is same and is equal to r, then G is called an *r-regular graph*. Let $v_1, v_2, \ldots, v_n$ be the vertices of G and let $d_i = deg_G(v_i)$. The *degree exponent matrix* of a graph G is an $n \times n$ matrix $DE(G) = [de_{ij}]$, whose elements are defined as

$$de_{ij} = \begin{cases} d_i^{d_j} & \text{if } i \neq j \\ 0 & \text{if } i = j. \end{cases}$$

Let I be the identity matrix and J be the matrix whose all entries are equal to 1. The *degree exponent polynomial* of a graph G is defined as

$$P_{DE(G)}(\gamma) = \det(\gamma I - DE(G)).$$

The eigenvalues of the matrix $DE(G)$, denoted by $\gamma_1, \gamma_2, \ldots, \gamma_n$ are called the *degree exponent eigenvalues* of G and their collection is called the *degree exponent spectra* of G. It is easy to see that if G is an r-regular graph, then $DE(G) = r^r J - r^r I$. Therefore, for an r-regular garph G of order n,

$$P_{DE(G)}(\gamma) = [\gamma - r^r(n-1)] [\gamma + r^r]^{n-1}. \tag{1}$$

2 Degree exponent polynomial of graphs obtained by graph operations

In this section we obtain the degree exponent polynomial of graphs obtained by some graph operations.

The *line graph* $L(G)$ of a graph G is a graph whose vertex set is in one-to-one correspondence with the edge set of G and two vertices of $L(G)$ are adjacent if and only if the corresponding edges are adjacent in G [7].

The *k-th iterated line graph* of G is defined as $L^k(G) = L(L^{k-1}(G))$, $k = 1, 2, \ldots$, where $L^0(G) \equiv G$ and $L^1(G) \equiv L(G)$.

Theorem 2.1 *Let G be an r-regular graph with n vertices and let n_k be the*