



Graceful labeling of some zero divisor graphs

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Abstract

This paper contains some results on graceful labeling of some zero divisor graphs. We prove that the zero divisor graph of \mathbb{Z}_n , the commutative ring of integers modulo n , is graceful if $n = pq$ or $4p$ or $9p$, where p and q both are prime numbers.

Keywords: Zero divisor graph, labeling, graceful graph.

1 Introduction and Preliminaries

A non empty set R is said to form a ring with respect to two binary compositions, addition (+) and multiplication (.) defined on it, if the following

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conditions are satisfied, $(R, +)$ is a commutative group, (R, \cdot) is a semigroup, left distributive law and right distributive law both hold in R .

A zero divisor is a non zero element ‘ a ’ of some structure S such that there is another non zero element ‘ b ’ $\in S$ with $ab = 0$. In [5], Beck presented the idea of creating a graph out of a commutative ring. However, in 1999, Anderson and Livingston [4] showed how $Z(R)$ exhibited interesting graph-theoretic properties. The zero divisor graphs have been extensively studied by various authors, see for example [1,2,3,4]. If R be a commutative ring and $Z(R)$ be the set of the zero divisors of R , then the zero divisor graph of R denoted by $\Gamma(R)$, is the graph whose vertex set is $Z(R)$ and in which two vertices u, v are adjacent if $u \neq v$ and $uv = 0$.

Let G be a graph of order n and size m . A one to one function $f : V(G) \longrightarrow \{0, 1, 2, \dots, m\}$ is called a graceful labeling of G if the induced edge labeling $f' : E(G) \longrightarrow \{1, 2, 3, \dots, m\}$ defined by $f'(e) = |f(u) - f(v)|$ is bijective for each edge $e = uv$ of G . A graph G , which admits graceful labeling is called a graceful graph.

The concept of graceful labeling was introduced by Rosa [10] in 1960s. Since then many different types of graph labeling techniques have been investigated and over 1000 papers been published in this area. For an excellent survey on graceful and related labelings, we refer the reader to Gallian [7]. In the present paper, we have introduced gracefulnes of zero divisor graphs of a commutative ring \mathbb{Z}_n , if $n = pq$ or $4p$ or $9p$, where p and q both are prime numbers.

2 Main Results

We have the following theorem due to Golomb [8].

Theorem 2.1 *Every complete bipartite graph is graceful.*

The following are some basic results found in any textbook on ring theory [6].

Theorem 2.2 *If R be a commutative ring and $Z(R)$ be the set of the zero divisors of R and $Z^*(R) = Z(R) \cup \{0\}$. Then $Z^*(R)$ is finite if and only if either R is finite or an integral domain.*

Theorem 2.3 *In ring \mathbb{Z}_n , an element $\bar{a} \in \mathbb{Z}_n$, where $0 < a < n$ is a zero divisor if and only if $\gcd(a, n) > 1$.*

Theorem 2.4 [9] *Let v be a vertex of the zero divisor graph $\Gamma(\mathbb{Z}_n)$. Then, $\deg(v) = \gcd(n, v) - 1$, if $v^2 \neq 0$ and $\deg(v) = \gcd(n, v) - 2$, if $v^2 = 0$.*

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