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Some results on the Laplacian spectra of graphs with pockets

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Abstract

Let F, H_v be simple connected graphs on n and $m+1$ vertices, respectively. Let v be a specified vertex of H_v and $u_1, \ldots, u_k \in F$. Then the graph $G = G[F, u_1, \ldots, u_k, H_v]$ obtained by taking one copy of F and k copies of H_v, and then attaching the i-th copy of H_v to the vertex u_i, $i = 1, \ldots, k$, at the vertex v of H_v (identify u_i with the vertex v of the i-th copy) is called a graph with k pockets. In [2], Barik raised the question that 'how far can the Laplacian spectrum of G be described by using the Laplacian spectra of F and H_v?' and discussed the case when $\deg(v) = m$ in H_v. In this article, we study the problem for more general cases and describe the Laplacian spectrum. As an application, we construct new nonisomorphic Laplacian cospectral graphs from the known ones.

Keywords: Laplacian matrix, Laplacian spectrum, join, pockets.

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1 Introduction

Throughout this article we consider only simple graphs. Let $G = (V(G), E(G))$ be a graph with vertex set $V(G) = \{1, 2, \ldots, n\}$ and edge set $E(G)$. The *adjacency matrix* of G, denoted by $A(G)$, is the $n \times n$ matrix whose (i, j)-th entry is 1, if i and j are adjacent in G and 0, otherwise. The *Laplacian matrix* of G is defined as $L(G) = D(G) - A(G)$, where $D(G)$ is the diagonal degree matrix of G. It is well known that $L(G)$ is a symmetric positive semidefinite matrix. Throughout this paper the *Laplacian spectrum* of G is defined as $\sigma(G) = (\lambda_1(G), \lambda_2(G), \cdots, \lambda_n(G))$, where $\lambda_1(G) \leq \lambda_2(G) \leq \cdots \leq \lambda_n(G)$ are the eigenvalues of $L(G)$ arranged in nondecreasing order. For any graph G, $\lambda_1(G) = 0$ afforded by the all ones eigenvector $\mathbf{1}$. An extensive literature is available on works related to Laplacian matrices and their spectra. Interested readers are referred to [1,9] and the references therein.

Two graphs are said to be *Laplacian cospectral* if they share same Laplacian spectrum. Haemer and Spence [6] enumerated the numbers for which there are at least two graphs with the same Laplacian spectrum and gave some techniques for their construction.

From literature we find many operations defined on graphs such as disjoint union, complement, join, product (Cartesian, direct, strong, lexicographic product etc.), corona and many variants of corona (like edge corona, neighbourhood corona, edge neighbourhood corona) etc. For such operations often it is possible to describe the Laplacian spectrum of the resulting graph using the Laplacian spectra of the corresponding constituting graphs, see [4,5] for reference. This enables one to visualize a complex network in terms of small simple recognizable graphs whose Laplacian spectra is easily computable. It is always interesting for researchers in spectral graph theory to define some new graph operations such that the Laplacian spectra of the new graphs produced can be described using the Laplacian spectra of the constituent graphs.

If $F = (V(F), E(F))$ and $H = (V(H), E(H))$ are two graphs on disjoint sets of m and n vertices, respectively, their union is the graph $F + H = (V(F) \cup V(H), E(F) \cup E(H))$, and their join is $F \vee H = (F^c + H^c)^c$, the graph on $m + n$ vertices obtained from $F + H$ by adding new edges from each vertex of F to every vertex of H.

The following result which describes the Laplacian spectrum of join of two graphs is often used in the next sections.

Theorem 1.1 ([8], Theorem 2.1) *Let F, H be graphs on disjoint sets of m, n, vertices respectively, and $G = F \vee H$. Let $\sigma(F) = (\lambda_1, \lambda_2, \ldots, \lambda_m)$ and $\sigma(H) = (\mu_1, \mu_2, \ldots, \mu_n)$. Then $0, n+\lambda_2, \cdots, n+\lambda_m, m+\mu_2, \cdots, m+\mu_n, m+n \in \sigma(G)$.*