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Structure of wheel-trees with colourings and domination numbers

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Abstract

The purpose of this paper is to introduce the wheel-trees, which are the natural extensions of 4-trees studied in [1,4] and also establish the properties and characterizations of the wheel-trees $G\langle W_k\rangle$ for $k \geq 6$. Moreover, the chromatic number, centrality, domination and Roman domination numbers of the wheel-trees of order ≥ 6 are determined.

Keywords: Cycle, Path, Tree, Wheel, Domination number, Colouring.

1 Introduction

We shall consider only finite, simple and undirected graphs. We follow the terminology of [3]. Given a graph G, $V(G)$ and $E(G)$ denote the sets of vertices

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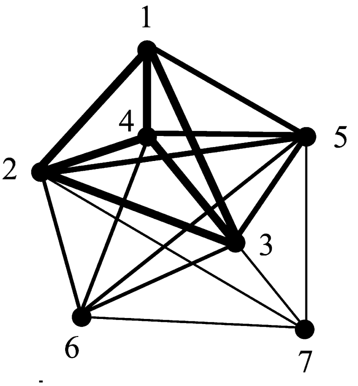

Fig. 1. W_4-tree of order 7

and edges of G, respectively. The graph K_1+C_{n-1} for $n \geq 4$, is a *wheel* and is denoted by W_n. Further, C_{n-1} is the *rim of* W_n and the vertex corresponding to K_1 is the *hub of* W_n. A *tree* is usually defined by the following recursive construction rule :

Step 1. A single vertex K_1 is a tree.

Step 2. Any tree of order $n \geq 2$, can be constructed from a tree Q of order $(n-1)$ by inserting an nth - vertex and joining it to any vertex of Q.

The above definition, motivates to extend the tree-construction rule by allowing the base of the recursive growth to be any wheel. It is natural that a connected graph, which is not a tree possesses a structure that look like a tree [6]. This kind of resemblance is actually reflected by constructing the new family of graphs, whose recursive growth just starts from any given wheel. With this view, for every wheel, there is associated another graph, we call *wheel-tree*, that is constructed as follows.

Definition 1.1 Let W_k be any given wheel of order k. A W_k*-tree* (or a wheel-tree), denoted by $G\langle W_k\rangle$, is a graph that can be obtained by the following recursive construction rule :

(i) W_k is the smallest W_k-tree of order k.

(ii) To a W_k-tree $G\langle W_k\rangle$ of order $n \geq k$, insert a new $(n+1)th-$vertex and join it to any set of k distinct vertices : $v_{i_1}, v_{i_2}, \ldots, v_{i_k}$ of $G\langle W_k\rangle$, so that the induced subgraph $\langle\{v_{i_1}, v_{i_2}, \ldots, v_{i_k}\}\rangle$ is isomorphic to W_k.

For example, W_4-tree of order 7 is shown in Fig. 1. Now, the object of this paper is to study the new family of W_k-trees and also, obtain their basic properties and characterizations. The interest in this class of graphs is motivated